THE INDIAN TRADITION OF SCIENCE: AN INTRODUCTORY OVERVIEW

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ABSTRACT

We present an overview of the development of science in India from ancient times to the modern period. We shall focus mainly on the disciplines of Mathematics and Astronomy, and briefly touch upon the important disciplines of Linguistics and Medicine. In all these disciplines, a high level of systematization was achieved at a fairly early stage and this was followed by significant developments all through the classical and the medieval periods. Indian contributions have also had major impact on the development of sciences in other civilisational areas.

Apart from highlighting such achievements, we shall also briefly discuss the foundational methodology of Indian sciences. Traditionally, such issues have been dealt with in the detailed Bhāṣyas or commentaries, which continued to be written till recent times and played a vital role in the traditional scheme of learning. Here, we shall focus on the pragmatic and open-ended approach to scientific theorisation as expounded in the canonical scientific texts of India, which is strikingly different from the ideal of absolutely true universal laws which has dominated the Greco-European scientific tradition.

OUTTLINE

- Indian Science in the Ancient (prior to 500 BCE) and Early Classical (500 BCE to 600 CE) Periods
- Indian Science in the Later Classical Period (600-1250)
- Indian Science in the Medieval Period (1250-1750)
- Indian Science in the Modern Period (Post 1750)
- Indian Approach to Science

INDIAN SCIENCE IN THE ANCIENT (PRIOR TO 500 BCE) AND EARLY CLASSICAL (500 BCE TO 600 CE) PERIODS

DEVELOPMENT OF SCIENCE OF LANGUAGE

Ancient and Early Classical Period (prior to 500 BCE)

- Padapāṭha, Kramapāṭha, Jaṭāpāṭha etc., of the Vedas
- Prātiśākhya & Śikṣā Texts, Yāska's *Nirukta* (etymology)
- Śākalya, Āpiśali, Indra, Kāśakṛtsna, Śākaṭāyana, Vyāḍi, etc.
- Pāṇini (prior to 500 BCE): *Aṣṭādhyāyī-Sūtrapāṭha*, *Dhātupāṭha*, *Gaṇapāṭha*, etc.
- Kātyāyana: *Vārttika*,
- Patañjali (c.150 BCE): *Mahābhāṣya*
- Śarvavarman: *Kātantra-vyākaraṇa*
- Candragomin (c.450 CE): *Cāndra-vyākaraṇa*
- Devanandin (c.450): *Jainendra-vyākaraṇa*
- Bhartṛhari (c.450): *Vākyapadīya*, *Mahābhāṣyadīpikā*
- Jayāditya, Vāmana (c. 600): *Kāśikāvṛtti*

Grammars of Other Languages

- *Pāli-vyākaraṇa* of Kātyāyana
- *Tolkāppiyam* of Tolkāppiyar

VARṆASAMĀMNĀYA: CLASSIFICATION OF SOUNDS

"The Padapāṭha discovered the differences between sentences, words, stems, pre- and suffixes, roots, etc., ...

The *Prātiśākhya* added an almost perfect analysis of the sounds of language into vowels, consonants, semi-vowels, stops, dentals, velars, nasals, etc. They placed these sounds in a two-dimensional configuration, developed from the square or *varga* of five-by-five series ...

Like Mendelejev's Periodic System of Elements, the *varga* system was the result of centuries of analysis."

Frits Staal, "The Science of Language", in G. Flood ed., *Blackwell Companion to Hinduism*, 2003, p.352.

VARṆASAMĀMNĀYA: THE INDIAN ALPHABET

sparśa (Plosive)									*anunāsika* (Nasal)	*antastha* (Approximant)		*ūṣma/saṃghaṣhrī* (Fricative)			
Voicing →	*aghoṣa*				*ghoṣa*								*aghoṣa*		*ghoṣa*	
Aspiration→	*alpaprāṇa*		*mahāprāṇa*		*alpaprāṇa*		*mahāprāṇa*		*alpaprāṇa*				*mahāprāṇa*			
kaṇṭhya **(Guttural)**	क	ka /k/	ख	kha /kʰ/	ग	ga /g/	घ	gha /gʱ/	ङ	ṅa /ŋ/					ह	ha /ɦ/
tālavya **(Palatal)**	च	ca /c, t͡ʃ/	छ	cha /cʰ, t͡ʃʰ/	ज	ja /ɟ, d͡ʒ/	झ	jha /ɟʱ, d͡ʒʱ/	ञ	ña /ɲ/	य	ya /j/	श	śa /ɕ, ʃ/		
mūrdhanya **(Retroflex)**	ट	ṭa /ʈ/	ठ	ṭha /ʈʰ/	ड	ḍa /ɖ/	ढ	ḍha /ɖʱ/	ण	ṇa /ɳ/	र	ra /r/	ष	ṣa /ʂ/		
dantya **(Dental)**	त	ta /t̪/	थ	tha /t̪ʰ/	द	da /d̪/	ध	dha /d̪ʱ/	न	na /n/	ल	la /l/	स	sa /s/		
oṣṭhya **(Labial)**	प	pa /p/	फ	pha /pʰ/	ब	ba /b/	भ	bha /bʱ/	म	ma /m/	व	va /w, ʋ/				

VARṆASAMĀMNĀYA: THE INDIAN ALPHABET

"It is not surprising that this classification was taken into account when the first Indian scripts evolved, but it went much further and served, for millennia to come, as a sound foundation for most of the numerous scripts and writing systems of south, southeast and east Asia – from Kharosthi, Khotanese, Tibetan, Nepali, and all the modern scripts of India (except the Urdu/Persian) to Sinhalese, Burmese, Khmer, Thai, Javanese, and Balinese. In south and southeast Asia, the *shapes* of earlier Indian syllables inspired some of these inventions, but it is the *system* of classification that was of enduring significance wherever it became known. In east Asia, the bastion of Chinese characters could not adapt it; but in Japan it led to the creation of the *hiragana* and *katakana* syllabaries during the Heian period (794–1185), and in Korea it inspired the world's most perfect script, *han'gul*, developed in 1444 by a committee of scholars appointed by the emperor Sejong. All these Asian scripts are a far cry from the haphazard jumble of the "ABC" and the countless spelling problems that result from it in English and other modern languages that use the alphabet."

Frits Staal, "The Science of Language", in G. Flood ed., *Blackwell Companion to Hinduism*, 2003, p. 357.

The first *Vārtika* states the purpose of Grammar:

> **अथ शब्दानुशासनम् ।**
>
> अनुशासनं प्रकृतिप्रत्ययविभागेन व्युत्पादनम् तद्व्याकरणेन साक्षात्क्रियत इति साक्षात्प्रयोजनम् । [अन्नम्भट्टीय प्रदीपोद्योतव्याख्या]
>
> "Now, the instruction of utterances
>
> Instruction, namely generation (of utterances) making use of the division into Prakṛti and Pratyaya, this is realised by Grammar, and that is its direct purpose."
>
> [*Annaṃbhaṭṭīya Pradīpodyotavyākhyā*]

In *Aṣṭādhyāyī,* Pāṇini achieves a complete characterisation of the Sanskrit language as spoken at his time, and also specifies the way it deviated from the Sanskrit of the Vedas.

PĀṆINI'S AṢṬĀDHYĀYĪ

The grammar has four distinct components:

1. *Śivasūtras*: The inventory of phonological segments.
2. *Aṣṭādhyāyī*: A system of about 4,000 grammatical rules.
3. *Dhātupāṭha*: A list of about 2,000 verbal roots.
4. *Gaṇapāṭha*: A list of 261 lists of lexical items.

Using the Sūtras of Pāṇini, along with the list of root words (*Dhātupāṭha*) and the list of lexical items (*Gaṇapāṭha*), it is possible to generate all possible valid utterances in Sanskrit.

All other disciplines have been deeply influenced by the ingenious symbolic and technical devices, the recursive and generative formalism, and the system of conventions governing rule application and rule interaction found in *Aṣṭādhyāyī*.

PĀṆINI'S AṢṬĀDHYĀYĪ

"Pāṇini has composed a list of formulae called *sūtra*...serving to form words and sentences from a given material of minimal elements...It comprises both lists of primary elements, and a program for the combination of these elements. These elements are the phonemes, the roots, group of words sharing a grammatical feature, morphemes (suffixes) having a meaning ...

The program is made up of operating rules as well as conventions necessary for the application of the rules. It is composed in a true meta-language very apt to its purpose, achieving the maximum brevity, which makes it easy to memorize, and is the first and foremost example of the formalization of the technical exposition in the universal history of sciences. Because of its practical objective and form, it cannot be compared with a systematic grammar of a European type. By contrast, its resemblance to a modern computer program is striking. "

[P. S. Filliozat: *The Sanskrit Language: An Overview*, Indica Books, Varanasi 2000 (French Edition 1992), p. 24]

ŚIVA-SŪTRAS AND PRATYĀHĀRAS

१ अइउण्। २ ऋलृक्। ३ एओङ्। ४ ऐऔच्। ५ हयवरट्। ६ लण्। ७ ञमङणनम्। ८ झभञ्। ९ घढधष्। १० जबगडदश्। ११खफछठथचटतव्। १२ कपय्। १३ शषसर्। १४ हल्॥

Each Sūtra has a set of Varṇas followed by a marker (ण्, क्, ङ्, च्, ट्, etc) called the इत् Varṇa [एषाम् अन्त्या इतः]

Pratyāhāras are formed by any of the Varṇas and an इत् which follows it. The Pratyāhāra then stands for the class of Varṇas enclosed by them except for the intervening इत् *varṇas*.

अक् stands for {अ,इ,उ,ऋ,लृ }. इक् stands for {इ,उ,ऋ,लृ }

अच् stands for all the vowels. हल् stands for all the consonants.

In this way about 300 Pratyāhāras are possible. Pāṇini uses 42 of them.

Recent studies show that the Śiva-sūtras give an optimal encoding for these 42 partially ordered subsets of Sanskrit sounds.

TYPES OF RULES IN AṢṬĀDHYĀYĪ

Pāṇini's Sūtras are mainly of the following types:

- Vidhi-sūtra: Operational rules.
- Saṃjñā-sūtra: Rules which introduce class names and establish conventions regarding the use of terms.
- Adhikāra-sūtra: Headings.
- Paribhāṣā-sūtra: Metarules, which serve to interpret and regulate other rules. They regulate the operations specified in the Vidhi-sūtras.

METARULES IN AṢṬĀDHYĀYĪ

Examples of Paribhāṣā-sūtras:

- **Ṣaṣṭhīsthāneyogā** (1.1.49): Genitive designates ‘in place of’.
- **Tasminnitinirdiṣṭe pūrvasya** (1.1.66): Locative defines the right context.
- **Tasmādityuttarasya** (1.1.67)**:** Ablative defines the left context.
- **Yathāsaṃkhyamanudeśaḥ samānām** (1.3.10): For groups with the same number of elements, the corresponding elements are to be related in order.
- **Pūrvatrāsiddham** (8.2.1): [From now on every rule is regarded as] not having taken effect with reference to preceding ones.

CONTEXT SENSITIVE RULES OF AṢṬĀDHYĀYĪ

Phonological rules are typically of the form "sounds of class A are replaced by sounds of class B if they are preceded by sounds of class C and followed by sounds of class D", which in modern phonology is usually denoted as

$$A \rightarrow B/_{C\_\_D} \ . \tag{1}$$

In Pāṇinian style rule (1) becomes

$$A + \text{genitive}, B + \text{nominative}, C + \text{ablative}, D + \text{locative} \ . \tag{2}$$

Example: **Ikoyaṇaci** (6.1.77)

ikoyaṇaci (6.1.77)

iK stands for {**i, u, ṛ, ḷ**},

yaṆ stands for {**y, v, r, l**}

aC stands for all the vowels.

From 6.1.72, **saṃhitāyām** is carried forward. Thus the Sūtra provides that:

i, u, ṛ, ḷ →y, v, r, l before a vowel, in close contact

This gives

i + a → y + a, u+ a → v + a, and so on.

Akaḥ savarṇe dīrghaḥ (6.1.101) is an Apavāda-sūtra to the above, and gives:

i + i = ī, u+ u = ū, and so on.

PĀṆINI'S GRAMMAR ACCLAMED ALL OVER ASIA (c. 675)

"Even in the island of Pulo Condore (in the south) [in Vietnam] and in the country of Sūli (in the north) [in Uzbekistan], people praise the Sanskrit Sūtras [of Pāṇini], how much more then people of the Divine Land (China), as well as the Celestial Store House [of knowledge] (India), teach the real rule of the language!...

Grammatical science is called, in Sanskrit, Śabdavidyā, one of the five Vidyās; Śabda meaning voice and Vidyā science. The name for the general secular literature in India is Vyākaraṇa, of which there are about five works, similar to the Five Classics of the Divine Land (China)...

The Sūtra is the foundation of all grammatical science. This name can be translated by 'short aphorism', and signifies that important principles are expounded in a simplified form. It contains 1,000 Ślokas [32,000 syllables], and is the work of Pāṇini, a very learned scholar of old, who is said to have been inspired and assisted by Maheśvara-deva...Children begin to learn the Sūtra when they are eight years old, and can repeat it in eight months time."

J. Takakasu, *A Record of Buddhist Religion as Practised in India and the Malay Archipelago by I-Tsing (671-695)*, Oxford 1896, pp. 169-172. I-Tsing is said to have learnt Sanskrit in Śrīvijaya (Sumatra) for six months in 671.

DEVELOPMENT OF INDIAN ASTRONOMY & MATHEMATICS

Ancient Period (Prior to 500 BCE)

- Astronomy and Calendar in the age of Vedic Saṁhitās.
- Knowledge of Astronomy as revealed in ancient archaeological sites.
- *Vedāṅgajyotiṣa* (c.1350 BCE): Computation of the Tithi, Nakṣatra etc., which depend on the motion of the Sun and the Moon, based on a five year Yuga cycle.
- *Parāśarasaṁhitā, Vṛddhagargasaṁhitā.*
- *Śulvasūtras* (prior to 600 BCE): The oldest texts of geometry. Give procedures for the construction and transformation of geometrical figures and alters (Vedi) using rope (Rajju) and gnomon (Śaṅku).
- Some of the ancient astronomical Siddhāntas are also from this period.

JYOTIḤŚĀSTRA AS KĀLAVIDHĀNAŚĀSTRA

According to *Vedāṅgajyotiṣa* of Lagadha

यथा शिखा मयूराणां नागानां मणयो यथा।
तद्वद्वेदाङ्गशास्त्राणां ज्योतिषं मूर्धनि स्थितम्॥

Just like the combs of the peacock and the crest jewels of the serpents, so does astronomy stand at the head of the Vedāṅga-śāstras.

वेदा हि यज्ञार्थमभिप्रवृत्ताः कालानुपूर्व्या विहिताश्च यज्ञाः।
तस्मादिदं कालविधानशास्त्रं यो ज्योतिषं वेद स वेद यज्ञान्॥

The Vedas have indeed been revealed for the sake of Yajñas. And the Yajñas are to be performed at the specified times. Hence, one who knows this science of astronomy, the science of time, he knows the Yajñas.

JYOTIḤŚĀSTRA AS KĀLAVIDHĀNAŚĀSTRA

Bhāskarācārya II (c.1150 CE) also defines the purpose of the science of astronomy very similarly in his celebrated textbook *Siddhāntaśiromaṇi*:

वेदास्तावद्यज्ञकर्मप्रवृत्ता यज्ञाःप्रोक्तास्ते तु कालाश्रयेण।
शास्त्रादस्मात्कालबोधो यतः स्याद्वेदाङ्गत्वं ज्यौतिषस्योक्तमस्मात्॥

The commentator Nṛsiṁhadaivajña states that

कालशब्दोऽपि दिगाद्युपलक्षणार्थः

In other words, Astronomy is the science of determination of Dik, Deśa and Kāla: Direction, Location and Time. And these are determined by observing the motion of celestial bodies.

To cite Lagadha once again:

ज्योतिषामयनं कृत्स्नं प्रवक्ष्याम्यनुपूर्वशः।
विप्राणां सम्मतं लोके यज्ञकालार्थसिद्धये॥

INDIAN CALENDAR

The Vedic corpus presents clear evidence of the early evolution of the Indian Luni-Solar calendar. The notions of solar year, lunar month, intercalary month, six seasons, the 27 Nakṣatra division of the zodiac, and even the 18 year eclipse cycle, have all been traced to the Vedic literature.

The Indian calendar is based on observed astronomical phenomena and not on convention.

Sideral solar year: Time taken for the Sun to traverse from Meṣādi to Meṣādi (365.25636 civil days on the average). Different from the Tropical year, which is the time interval between successive crossings of vernal equinox (365.24219 days on the average).

The Solar year begins when the true Sun enters Meṣa-rāśi. The Luni-Solar year begins at the Amāvāsyā (conjunction of Sun and Moon) prior to that.

Lunar month: Time interval between successive conjunctions of the Sun and the Moon (average synodic period 29.5306 days). Divided into 30 Titihis. Adhikamāsa is a lunar month that does not include a Solar Saṅkrānti (transit of the Sun from one Rāśi to another).

FIVE YEAR YUGA CYCLE OF VEDĀṄGAJYOTIṢA

The *Vedāṅgajyotiṣa* of Lagadha is a brief manual (36 verses in the Ṛk recension and 43 in the Yajus recension) which presents simple algorithms for the computation of Tithi, Nakṣatra, seasons, solstices, the length of the day etc., based on a five year Yuga cycle beginning with winter solstice at Śraviṣṭhā (c.1350 BCE). [Currently in second Pāda of Mūlā]

The computational scheme of *Vedāṅgajyotiṣa* is based on the fact that 62 synodic periods (30 Tithis) and 67 sidereal periods of the Moon (27 Nakṣatras) are nearly equal to 1830 civil days. This is taken as a Yuga or a cycle of 5 Solar years of 366 civil days each.

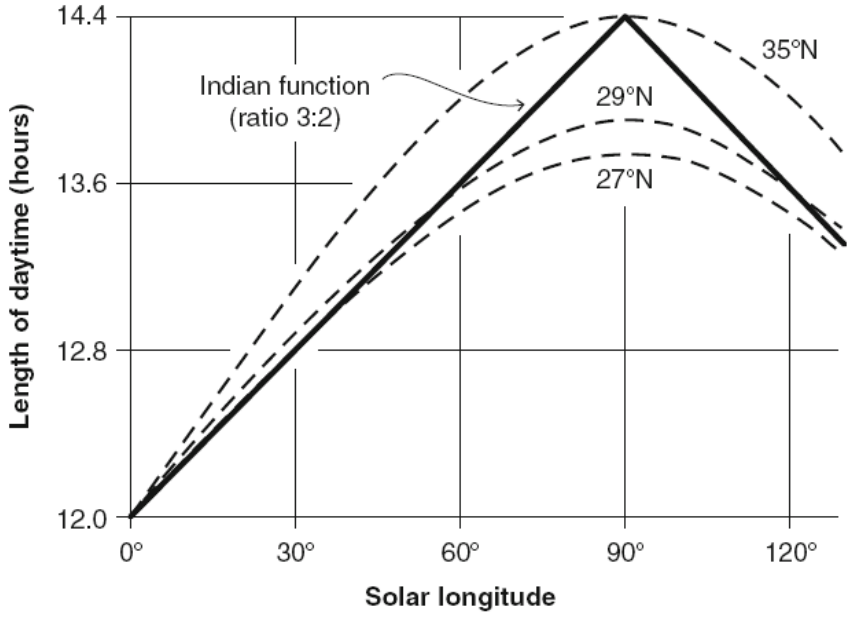
62 synodic periods = 62 x 29.530589 = 1830.897

67 sidereal period = 67 x 27. 321662 = 1830.551

VEDĀṄGAJYOTIṢA FORMULA FOR LENGTH OF DAYTIME

According to *Vedāṅgajyotiṣa*, the day is divided into 30 Muhūrtas. The duration of day time T in Muhūrtas on the n-th day starting from the winter solstice, is given by the formula

$$T = 12 + (2/61)n$$



This fits fairly well (except near solstices) for latitudes in North India.

BAUDHĀYANA-ŚULVASŪTRA (PRIOR TO 600 BCE)

- Units of measurement (Bhūmiparimāṇa)
- Marking directions and construction of a square of a given side (Samacaturaśra-karaṇa)
- Construction of a rectangle and isosceles trapezium of given sides
- Construction of √2 (Dvikaraṇī), √3 and √(1/3) times a given length
- 1.37: The square of the diagonal of a rectangle is the sum of the squares of its sides (Bhujā-Koṭi-Karṇa-Nyāya, the oldest theorem in Geometry)

दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत् पृथग्भूते कुरुतस्तदुभयं करोति

- 1.39: Construction of squares which are the sum and difference of two squares
- 1.41-42: Transforming a square into a rectangle, isosceles trapezium, isosceles triangle and a rhombus of equal area and vice versa
- 1.47: Approximate conversion of a square into a circle

  $r \approx (a/3)\,(2 + \sqrt{2})$. **[π ≈3.0883]**
- 1.50: An approximation for **√2** (dvikaraṇī)

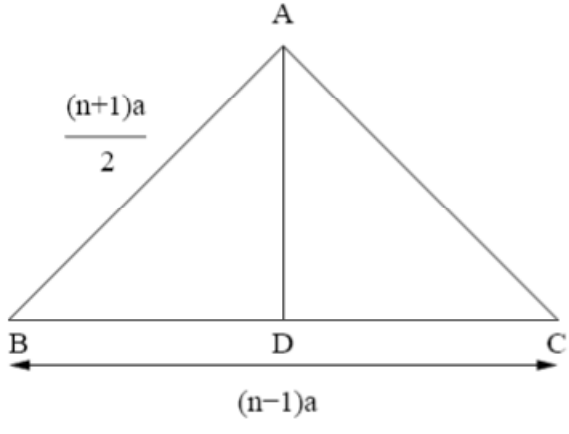
  √2 ≈1 + (1/3) + (1/3.4) - (1/3.4.34) = 1.4142156
- Positions, relative distances and areas of altars. Shapes of different altars and their construction.

To construct a square which is *n*-times a given square

यावत्प्रमाणानि समचतुरश्राण्येकीकर्तुं चिकीर्षेत् एकोनानि तानि भवन्ति तिर्यक् द्विगुणान्येकत एकाधिकानि । त्र्यस्रिर्भवति तस्येषुस्तत्करोति ।

[कात्यायनशुल्बसूत्रम् ६.७]

As many squares as you wish to combine into one, the transverse line will be one less than that. Twice the side will be one more than that. That will be the triangle. Its arrow (altitude) will produce that.



$$\begin{aligned} AD^2 &= AB^2 - BD^2 \\ &= \left[\frac{(n+1)a}{2}\right]^2 - \left[\frac{(n-1)a}{2}\right]^2 \\ &= na^2. \end{aligned}$$

DEVELOPMENT OF INDIAN ASTRONOMY & MATHEMATICS

Classical Period I (500 BCE – 600 CE)

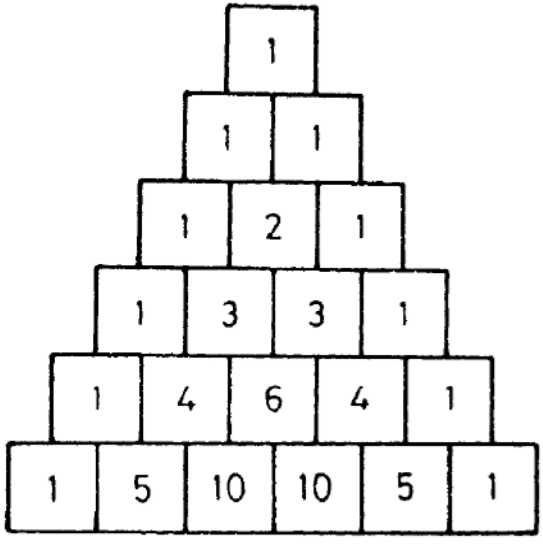
- Pervasive influence of the methodology of Pāṇini's *Aṣṭādhyāyī*
- Piṅgala's *Chandaḥsūtra* (c.300 BCE) and the development of binary representation and combinatorics
- Mathematical and Astronomical ideas in Bauddha and Jaina Texts
- The notion of zero and the decimal place value system
- *Candravākyas* of Vararuci
- *Paitāmahasiddhānta* of *Viṣṇudharmottarapurāṇa* (c. 400)
- Mathematics and Astronomy in *Āryabhaṭīya* (c.499 CE): The standard procedures in arithmetic, algebra, geometry and trigonometry, and the procedures for calculating planetary positions, eclipses, etc., are perfected by this time.
- Works of Varāhamihira (c. 505-587): *Pañcasiddhāntikā*, *Bṛhatsaṁhitā*, *Bṛhajjātaka*.
- Works of Bhāskara I (c.629): *Āryabhaṭīyabhāṣya*, *Mahābhāskarīya, Laghubhāskarīya*.

परे पूर्णमिति। (छन्दःशास्त्रम् ८. ३४)

Piṅgala's above Sūtra for computing the number of metrical forms of *n* syllables where *r* are Gurus (Lagakriyā) is too brief. Halāyudha, the tenth century commentator explains it as giving a recursive rule for the construction of the table of numbers which he refers to as the Meru-prastāra.

उपरिष्टादेकं चतुरस्रकोष्ठं लिखित्वा तस्यधस्तादुभयतोऽर्धनिष्क्रान्तं कोष्ठकद्वयं लिखेत्। तस्याप्यधस्तात्त्रयं तस्याप्यधस्ताच्चतुष्टयं यावदभिमतं स्थानमिति मेरुप्रस्तारः। तस्य प्रथमे कोष्ठे एकसंख्यां व्यवस्थाप्य लक्षणमिदं प्रवर्तयेत्। तत्र परे कोष्ठे यद्वृत्तसंख्याजातं तत् पूर्वकोष्ठयोः पूर्णं निवेशयेत्। तत्रोभयोः कोष्ठकयोरेकैकमङ्कं दद्यात् मध्ये कोष्ठे तु परकोष्ठद्वयाङ्कमेकीकृत्य पूर्णं निवेशयेदिति पूर्णशब्दार्थः। चतुर्थ्यां पङ्क्तावपि पर्यन्तकोष्ठयोरेकैकमेव स्थापयेत्। मध्यमकोष्ठयोस्तु परकोष्ठद्वयाङ्कमेकीकृत्य पूर्णं त्रिसंख्यारूपं स्थापयेत्।...

MERUPRASTĀRA OF PIṄGALA



The number of metrical forms with *r* Gurus (or Laghus) among the metres of *n*-syllables is the binomial coefficient ${}^{n}C_{r}$

The above passage of Halāyudha shows that the basic rule for the construction of the above table, is the recurrence relation

$${}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

DEVELOPMENT OF DECIMAL PLACE VALUE SYSTEM

- The Yajurveda-Saṁhitā talks of powers of 10 up to 10^{12} (Parārdha)
- The Upaniṣads talk of zero (Śūnya, Kha) and infinity (Pūrṇa).
- Pāṇini's *Aṣṭādhyāyī* uses the idea of zero-morpheme (Lopa)
- The Bauddha and Naiyāyika philosophers discuss the notions of Śūnya and Abhāva.
- Piṅgala's *Chandaḥśāstra* uses zero as a marker (*rupe śūnyam*)
- Philosophical works such as the works of Vasumitra (c.50 CE) and the *Vyāsabhāṣya* on *Yogasūtra* (3.1.3) refer to the way the same symbol acquires different connotations in the place value system.

  यथैका रेखा शतस्थाने शतं दशस्थाने दश एका च एकस्थाने यथा चैकत्वेपि स्त्री माता चोच्यते दुहिता च स्वसा चेति।
- Amongst the works whose dates are well established, decimal place value system occurs for the first time in the *Yavanajātak*a (c.270 CE) of Sphujidhvaja.
- The *Āryabhaṭīya* of Āryabhaṭa presents all the standard procedures of calculation based on the place value system.

DEVELOPMENT OF DECIMAL PLACE VALUE SYSTEM

An eighth century inscription in a Viṣṇu Temple in Gwalior, depicting the number 270 in decimal place value format. There are inscriptions of early 7th century in Southeast Asia which depict numbers in place value format.

INDIAN PLACE VALUE SYSTEM ACCLAIMED UNIVERSALLY

"I will omit all discussion of the science of the Hindus, a people not the same as Syrians, their subtle discoveries in the science of astronomy, discoveries that are more ingenious than those of the Greeks and the Babylonians; their computing that surpasses description. I wish only to say that this computation is done by means of nine signs. If those who believe because they speak Greek, that they have reached the limits of science should know these things, they should be convinced that there are also others who know something."

Syrian Monophysite Bishop Severus Sebokht (c.662)

"By the time I was ten I had mastered the Koran and a great deal of literature, so that I was marveled at for my aptitude….Now my father was one of those who has responded to the Egyptian propagandist (who was an Ismaili); he, and my brother too, had listened to what they had to say about the Spirit and the Intellect, after the fashion in which they preach and understand the matter. … Presently they began to invite me to join the movement, rolling on their tongues talk about philosophy, geometry, Indian arithmetic: and my father sent me to a certain vegetable-seller who used the Indian arithmetic, so that I might learn it from him."

From *The Autobiography* of the Islamic Philosopher Scientist Ibn Sina (980-1037)

INDIAN PLACE VALUE SYSTEM ACCLAIMED UNIVERSALLY

"It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to all computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of this achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity."

Pierre-Simon Laplace

"To what height would science now have been if Archimedes made that discovery [place value system]!"

Carl Friedrich Gauss

GAṆITA: INDIAN MATHEMATICS OF COMPUTATION

गण्यते संख्यायते तद् गणितम्। तत्प्रतिपादकत्वेन तत्संज्ञं शास्त्रमुच्यते।

As noted by Gaṇeśa Daivajña, in his commentary *Buddhivilāsinī* (c.1540) on *Līlāvatī* (c.1150), Gaṇita (Indian Mathematics) is the science (art) of computation. Indian Mathematical Texts give rules to describe systematic and efficient procedures of calculation.

The modern word **algorithm** derives from the medieval word 'algorism' which referred to the Indian methods of calculation based on the place value system. The word 'algorism' itself is a corruption of the name of the Central Asian mathematician al Khwarizmi (c.825) whose *Hisab al Hind* (Indian Method of Calculation) was the source from which the Indian methods of calculation reached the Western world.

Here is an ancient rule for squaring as cited by Bhāskara I (c.629 AD), uses $n(n\text{-}1)/2$ multiplications for squaring an n-digit number.

अन्त्यपदस्य वर्गं कृत्वा द्विगुणं तदेव चान्त्यपदम्।
शेषपदैराहन्यात् उत्सार्योत्सार्य वर्गविधौ॥

In the process for calculating the square, the square of the last digit is found (and placed over it). The rest of the digits are multiplied by twice the last digit (and the results placed over them). Then (omitting the last digit), moving the rest by one place each, the process is repeated again and again.

An Example: To calculate 125^2

1	5	6	2	5	
				25	$5^2 = 25$
		4	20		$2^2 = 4,\ 5.2.2 = 20$
1	4	10			$1^2 = 1,\ 2.2.1 = 4,\ 5.2.1 = 10$
1	2	5			

GAṆITAPĀDA OF ĀRYABHAṬĪYA (c.499)

Āryabhaṭīya consists of 108 verses in all (*Āryaṣṭaśatī* in Āryā metre), of which the first 33 constitute the section on Mathematics (*Gaṇitapāda*), the next 25 verses are on the computation of time (*Kālakriyāpāda*) and the last 50 are on spherics (Golapāda). Apart from these, there are 13 verses of the *Gītikāpāda* in (*Gītikā* metre) which give all the parameters associated with the motion of the Sun, Moon and the planets, and the sine table.

The following are the topics dealt with in the *Gaṇitapāda* of *Āryabhaṭīya*

- Saṅkhyāsthāna: Place values.
- Vargaparikarma, Ghanaparikarma: Squaring and cubing.
- Vargamūlānayana: Obtaining the square-root.
- Ghanamūlānayana: Obtaining the cube-root.
- Area of a triangle and volume of an equilateral tetrahedron.
- Obtaining the area of a circle, volume of a sphere
- Obtaining the area of a trapezium.
- Chord of a sixth of the circumference.
- Approximate value of the circumference ($\pi \approx 3.1416$).
- Jyānayana: Computing table of Rsines from the formula for second-order Rsine-differences.

- Chāyā-karma: Obtaining shadows of gnomons.
- Karṇānayana: Theorem on the square of the diagonal.
- Śarānayana: Arrows of intercepted arcs.
- Śreḍhī-gaṇita: Summing an AP, finding the number of terms, obtaining repeated summations.
- Varga-ghana-saṅkalanāyana: Obtaining the sum of squares and cubes of natural numbers.
- Mūlaphalānayana: Interest and principal.
- Trairāśika: Rule of three.
- Bhinna-parikarma: Arithmetic of fractions.
- Pratiloma-karaṇa: Inverse processes.
- Samakaraṇa-uddeśaka-pradarśana: Linear equation with one unknown.
- Yogakālānayana: Meeting time of two bodies.
- Kuṭṭākāra-gaṇita: Solution of linear indeterminate equation.

Thus, by the time of *Āryabhaṭīya*, Indian mathematicians had systematised most of the basic procedures of arithmetic, algebra, geometry and trigonometry that are generally taught in schools to-day, and many more that are more advanced (such as Kuṭṭaka and sine-tables) and are of importance in astronomy.

PLANETARY MODEL OF ĀRYABHAṬA

Verses 3, 4 of the *Gītikāpāda* give the mean revolution of the planets in a Mahāyuga of 43,20,000 years.

Planet	Revolutions in a Mahāyuga	Sidereal Period (days)	Modern Period (days)
Sun	43,20,000	365.25868	365.25636
Moon	5,77,53,336	27.32167	27.32166
Moon's Apogee	4,88,219	3231.98708	3232.37543
Moon's Node	2,32,226	6794.74951	6793.39108
Śīghrocca of Mercury	1,79,37,020	87.96988	87.9693
Śīghrocca of Venus	70,22,288	224.69814	224.7008
Mars	22,96,824	686.99974	686.9797
Jupiter	3,64,224	4332.27217	4332.5887
Saturn	1,46,564	10766.06465	10759.201

PLANETARY MODEL OF ĀRYABHAṬA

The planetary theory as discussed in the few verses of *Kālakriyāpāda* of *Āryabhaṭīya* is too cryptic; the commentary of Bhāskara I (c.630 AD) gives a detailed exposition.

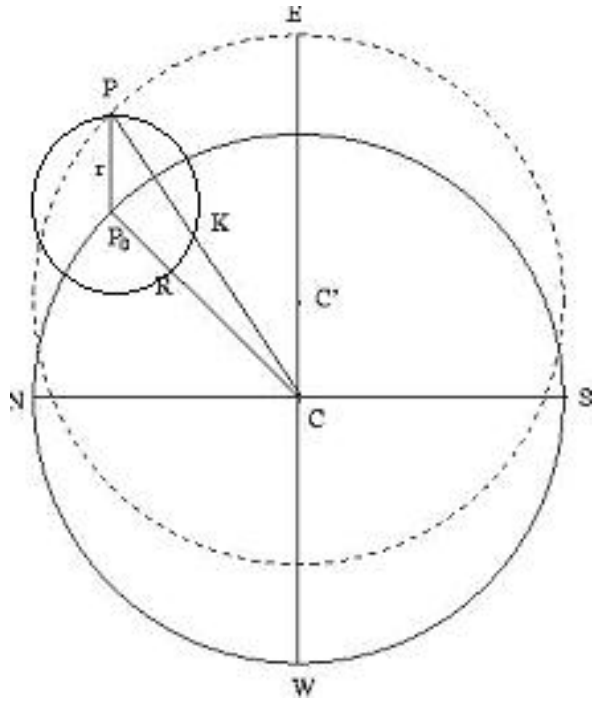
The procedure for calculating the geo-centric longitudes of the five planets, Mercury, Venus, Mars, Jupiter and Saturn involves essentially the following steps. First, the mean longitude (called the Madhyama-graha) is calculated for the desired day.

Then two corrections namely the Manda-saṃskāra and Śīghra-saṃskāra are to be applied to the mean planet. The Madhyama-graha corresponds to the mean-heliocentric planet. The Manda-correction corresponds to the equation of centre giving the true heliocentric planet. The Śīghra-saṃskāra corresponds to the process of conversion of the heliocentric longitude to the geocentric longitude.

In the case of Mercury and Venus, the mean Sun is taken as the mean planet and the equation of centre is applied to it – a feature common to all the ancient planetary theories (Indian, Greco-European & Islamic).

PLANETARY MODEL OF ĀRYABHAṬA

The Manda-correction is given by a variable epicycle model.



$$R\sin(P-M) = (r/K)\, R\sin(M-U) = (r_0/R)\, R\sin(M-U)$$

The actual orbit of the planet may be seen to be an oval – the first non-circular orbit in the history of astronomy.

The Manda-correction will be the same as the Keplerian equation of centre to the first order in eccentricity, if we have $(r_0/R) = 2e$

Planet	*Āryabhaṭīya* r_0/R	Modern $2e$
Sun	0.038	0.034
Moon	0.088	0.110
Mercury	0.075	0.412
Venus	0.038	0.014
Mars	0.200	0.186
Jupiter	0.094	0.096
Saturn	0.138	0.112

In the case of the five planets, apart from the Manda-correction there is the *śīghra*-correction, which is also given by an epicycle model.

The Śīghra-correction converts the heliocentric to geocentric longitudes if the ratio of the radius of the epicycle to that of the concentric, r_s/R, is equal to the mean ratio of the Sun-Planet and Earth-Sun distances in the case of interior planets, and the ratio of the Earth-Sun and Sun-Planet distances for exterior planets.

Planet	*Āryabhaṭīya*	Modern
Mercury	0.361 to 0.387	0.387
Venus	0.712 to 0.737	0.723
Mars	0.637 to 0.662	0.656
Jupiter	0.187 to 0.200	0.192
Saturn	0.100 to 0.113	0.105.

Thus we see that the Āryabhaṭa model is fairly accurate in capturing both the equation of centre and the transformation from heliocentric longitudes to the geocentric longitudes in the case of an exterior planet.

In the case of an interior planet, the Manda-correction or equation of centre for an interior planet is wrongly applied to the longitude of the mean Sun instead of the mean heliocentric longitude of the planet. The Śīghra-correction, then, merely gives an approximation to the geocentric longitude of mean heliocentric planet.

However, while computing the latitude of a planet, Āryabhaṭa prescribes that in the case of exterior planets it is the Manda-corrected mean planet (i.e., the true heliocentric planet) whose latitude is to be calculated; but, in the case of interior planets, it is the Śīghrocca and not the mean planet which should be made use of while calculating the latitude.

This prescription ensured that the traditional Indian planetary models were more accurate in their description of the latitudinal motion than the Greek, Islamic and the European planetary models till the time of Kepler.

DEVELOPMENT OF ĀYURVEDA

Ancient and Early Classical Period

- Āyurveda in the Vedas, especially *Atharvaveda*: physiological systems, the five Prāṇas, diseases caused by transgression of Ṛta (cosmic and individual moral order)
- Evidence of Āyurveda in ancient archaeological sites: Public and private baths, public sanitation,
- Āyurveda in the Bauddha Texts : *Tripiṭaka, Milindapanha, Mahāvagga, Jātakas*, etc.
- *Caraka-Saṁhitā* of Agniveśa, revised by Caraka and further revised by Dṛḍhabala (500 BCE – 400 CE) [More than sixty commentaries known]
- *Suśruta-Saṁhitā* (300 BCE): Divodāsa, Nāgārjuna, Suśruta [More than thirty commentaries known]
- *Bhelasaṃhitā, Kāśyapasaṃhitā, Hārītasaṃhitā, Nāvanītaka* (Bower Manuscript), etc.
- Vāgbhaṭa (c. 500 CE): *Aṣṭāṅgasaṅgraha*, *Aṣṭāṅgahṛdaya* [More than fifty commentaries known]
- Tirumūlar (c.600): *Tirumantiram* (Tamil Siddha text)

CARAKA ON THE PURPOSE OF ĀYURVEDA

The importance of health (Caraka, Sūtrasthāna 1.15)

धर्मर्थकाममोक्षाणामारोग्यं मूलमुत्तमम्। रोगास्तस्यापहर्तारःश्रेयसो जीवितस्य च॥

Health is at the very root of pursuing all the four ends of life: Dharma Artha, Kāma and Mokṣa. Diseases are destroyers of health, well-being and life itself.

Āyurveda: (Caraka Sūtrasthāna 1.42-43)

हिताहितं सुखं दुःखमायुस्तस्य हिताहितम्।
मानं च तच्च यत्रोक्तमायुर्वेदः स उच्यते॥
शरीरेन्द्रियसत्त्वात्मसंयोगो धारि जीवितम्।

Āyurveda is the science which discusses the beneficial and non-beneficial life, happy and unhappy life, and what is beneficial or not beneficial for them. It also discusses the span of life and life itself. Life is the combination of body, senses, mind and the Ātman, which maintains the body from decay.

Later Caraka says आयुर्वेदयतीत्यायुर्वेदः (Caraka,Sūtrasthāna 30.23).

CARAKA ON THE PURPOSE OF ĀYURVEDA

धातुसाम्यक्रिया चोक्ता तन्त्रस्यास्य प्रयोजनम्।
कालबुद्धीन्द्रियार्थानां योगो मिथ्या च नाति च॥
शरीरं सत्त्वसंज्ञं च व्याधीनामाश्रयो मतः ।
तथा सुखानां योगस्तु सुखानां कारणं सम:॥...
वायुः पित्तं कफश्चोक्तः शारीरो दोषसङ्ग्रहः।
मानसः पुनरुद्दिष्टो रजश्च तम एव च॥
प्रशम्यत्यौषधैर्पूर्वो दैवयुक्तिव्यपाश्रयैः।
मानसो ज्ञानविज्ञानधैर्यस्मृतिसमाधिभिः॥

The purpose of this treatise is to discuss the process of ensuring a balance of Dhātus. The cause of diseases of both (the body and mind) are in brief the wrong contact, non-contact and the excessive contact of time, mental faculties and the objects of senses. The body and the mind are the loci of diseases and of happiness; happiness is caused by balanced contact [of the above]. Vāyu, Pitta and Kapha are Doṣas of the body; Rajas and Tamas, of the mind. The bodily ones are ameliorated by therapies either based on Yukti or Daiva (Adṛṣṭa). The mental ones by knowledge of the Ātman and the Śāstras, forbearance, recollection and mind control. (Caraka: Sūtra 1.53-58)

CARAKA ON TYPES OF DISEASES AND THEREUPATICS

विपरीतगुणैर्देशमात्राकालोपपादितैः। भेषजैर्विनिवर्तन्ते विकाराः साध्यसम्मताः।
साधनं नत्वसाध्यानां व्याधीनामुपदिश्यते। [चरकसंहिता सूत्रस्थानम् १.६२-६३]

"The diseases which are curable are cured by medicines which have the contrary qualities, administered with due regard to place, amount and time. No method of cure is prescribed for diseases which are incurable."

त्रिविधमौषधमिति दैवव्यपाश्रयं युक्तिव्यपाश्रयं सत्त्वावजयश्च। तत्र दैवव्यापाश्रयं मन्त्रौषधिमणिमङ्गलबल्युपहारहोमनियमप्रायश्चित्तोपवासस्वस्त्ययनप्रणिपातगमनादि युक्तिव्यपाश्रयं पुनराहारौषधद्रव्याणां योजना सत्त्वावजयः पुनरहितेभ्योऽर्थेभ्यो मनोनिग्रहः। [चरकसंहिता सूत्रस्थानम् ११.५४]

"It is said that there are three kinds of therapeutics: Those based on Yukti, based on Daiva, and those based on the control of mind. There, the therapies based on Daiva (Adṛṣṭa or Gods) are Mantra, talismans, gems, Bali offerings, oblations, following the Niyamas, Prāyaścitta, fasts, auspicious hymns, prostration to gods, pilgrimage, etc. Then, the therapeutics based on Yukti is by the administration of proper food [and activities] and medicinal drugs [based on Yukti]. And therapeutics based on control of mind is by reigning in the mind away from harmful objects."

SUŚRUTA ON THE PURPOSE OF ĀYURVEDA

Suśruta and others approach Kaśīrāja Divodāsa Dhanvantari (Suśruta Sūtrasthāna 1.4):

> भगवन् शारीरमानसागन्तुभिर्व्याधिभिर्विविधवेदनाभिघातोपहतान् सनाथान्यपि अनाथवद् विचेष्टमानान् विक्रोशतश्च मानवानभिसमीक्ष्य मनसि नः पीडा भवति तेषां सुखैषिणां रोगोपशमनार्थमात्मनश्च प्राणयात्रार्थं प्रजाहितहेतोरायुर्वेदं श्रोतुमिच्छामः।
>
> "Oh exalted one! We are pained to see men who are afflicted with bodily, mental and exogenous diseases causing pain and injury, and who are crying and moving restlessly like orphans even though they are well protected. We would like to learn Āyurveda in order to ameliorate the diseases of those who desire happiness, for our own healthy living, and for the welfare of the people."

Dhanvantari responds as follows (Suśruta Sūtrasthāna 1.14-16):

> वत्स सुश्रुत इह खल्वायुर्वेदप्रयोजनं व्याध्युपसृष्टानां व्याधिपरिमोक्षः स्वस्थस्य रक्षणं च। आयुरस्मिन् विद्यते अनेनवाऽऽयुर्विन्दतीत्यायुर्वेदः। तस्मादङ्गवरमाद्यं प्रत्यक्षागमानुमानोपमानैरविरुद्धमुच्यमानमुपधारय।

SUŚRUTA ON THE PURPOSE OF ĀYURVEDA

"Dear Suśruta! Here, the purpose of Āyurveda is indeed to render those who are under the spell of diseases free from all diseases, and to protect those who are healthy. Āyurveda is the science of life and also for prolonging it. Hence learn this first and foremost part (of Āyurveda) which is being taught and which is not in contradiction with Pratyakṣa (perception), Āgama (authoritative text or tradition), Anumāna (reason) and Upamāna (analogy) [the means of acquiring valid knowledge]."

The commentator Dalhaṇa explains:

आगमस्य प्रत्यक्षफलत्वात् वरीयस्त्वं तेनानुमानात् पूर्वं निर्दिष्टवान्।

"The Āgama (authoritative text or tradition) is of a higher status because of its all too visible results and therefore the Ācārya has stated it prior to Anumāna (inference)."

HEALTHCARE IN INDIA C.400 CE

"The cities and towns of this country [Magadha] are the greatest of all the middle kingdom. The inhabitants are rich and prosperous, and vie with one another in the practise of benevolence and righteousness... The heads of the Vaiśya families in them [all the kingdoms of North India] establish in the cities houses for dispensing charity and medicine. All the poor and destitute in the country, orphans, widowers and childless men, maimed people and cripples, and all who are diseased go to these houses, and are provided with every kind of help, and the doctors examine their diseases. They get the food and medicines which their cases require, and are made to feel at ease; and when they are better, they go away of themselves."

Faxian account translated in A. J. Legge, *A Record of Buddhist Kingdoms; Being an Account by the Chinese Monk Faxian of his Travels in India and Ceylon (399-414) in Search of Buddhist Books of Discipline*, Oxford 1886, p.79

There is a detailed description of a hospital in *Carakasaṁhitā Sūtrasthāna* 15.1-7.

INDIAN SCIENCE IN THE LATER CLASSICAL PERIOD (600-1250)

Later Classical Period (c.600-1250)

- Jinendrabuddhi (c.900): *Kāśikāvivaraṇa-pañjikā* or *Nyāsa*
- Kaiyaṭa (c. 900): *Mahābhāṣya-pradīpa*
- Haradatta (c. 1000): *Padamañjarī*
- Dharmakīrti (c.1000): *Rūpāvatāra*
- Hemacandra (c. 1100): *Siddhahaimacandra*, etc
- Vopadeva (c.1250): *Mudgdhabodha*

Grammars of Other Languages

- Tamil: *Vīrasolīyam* (c.1200), *Nannūl* (c.1250)
- Kannada: *Karnāṭaka-bhāṣābhūṣaṇa* (c.1100), *Śabdamaṇidarpaṇa* (c.1200)
- Telugu: *Āndhra-śabdacintāmaṇi* (c.1100), *Āndhrabhāṣābhūṣaṇa* (c.1250)
- Pali: *Kaccāyana-vyākaraṇa*, *Saddalakkhaṇa* (c. 1150)
- Prākṛta: *Prākṛta-prakāśa*, *Prākṛta-śabdānuśāsana* (c.1200)

Later Classical Period (600 CE – 1200 CE)

- Works of Brahmagupta: *Brāhmasphuṭasiddhānta* (c.628 CE) *Khaṇḍa-khādyaka* (c.665): Mathematics of zero and negative numbers. Development of algebra. Second order interpolation, etc.
- Parahita system of Haridatta (c. 683)
- Bakṣālī Manuscript (c. 7-8th century): Development of mathematical notation.
- Works of Śrīdhara, Lalla (c.750), Govindasvāmin (c.800)
- *Gaṇitasārasaṅgraha of* Mahāvīrācārya (c.850): First treatise solely devoted to mathematics.
- Works of Pṛthūdakasvāmin (c.860), Vaṭeśvara (c. 904), Muñjāla (c.932), Āryabhaṭa II (c.950), Śrīpati (c.1039) and Jayadeva (c.1050): Second correction to Moon. Equation of Time. Cakrvāla method, etc.
- Works of Bhāskarācārya II (c.1150): *Līlāvatī, Bījagaṇita*, and *Siddhānta-śiromaṇi*: They became the canonical texts of Indian mathematics and astronomy. Upapattis (proofs) in Bhāskara's *Vāsanābhāṣyas*.
- Mathematics in works of Prosody (such as *Vṛttajātisamuccaya*, *Vṛttaratnākara*), Music (such as *Saṅgītaratnākara*, etc), Architecture, etc.

MAHĀVĪRĀCĀRYA ON THE ALL-PERVASIVENESS OF GAṆITA (c.850 CE)

लौकिके वैदिके वापि तथा सामायिकेऽपि यः। व्यापारस्तत्रसर्वत्र संख्यानमुपयुज्यते॥
कामतन्त्रेऽर्थशास्त्रे च गान्धर्वे नाटकेऽपि वा। सूपशास्त्रे तथा वैद्ये वास्तुविद्यादिवस्तुषु॥
छन्दोऽलङ्कारकाव्येषु तर्कव्याकरणादिषु। कलागुणेषु सर्वेषु प्रस्तुतं गणितं परम्॥
सूर्यदिग्रहचारेषु ग्रहणे ग्रहसंयुतौ। त्रिप्रश्ने चन्द्रवृत्तौ च सर्वत्राङ्गीकृतं हि तत्॥
द्वीपसागरशैलानां संख्याव्यासपरिक्षिप: । भवनव्यन्तरज्योतिर्लोककल्पाधिवासिनाम्॥
नरकाणां च सर्वेषां श्रेणीबन्धेन्द्रकोत्कराः। प्रकीर्णकप्रमाणाद्या बुध्यन्ते गणितेन ते॥
प्राणिनां तत्र संस्थानमायुरष्टगुणादयः। यन्त्राद्याः संहिताद्याश्च सर्वे ते गणिताश्रयाः॥
बहुभिर्विप्रलापैः किं त्रैलोक्ये सचराचरे। यत्किंचिद्वस्तु तत्सर्वं गणितेनविना न हि॥

[महावीराचार्यविरचित-गणितसारसङ्ग्रहः १.९-१६]

MAHĀVĪRĀCĀRYA ON THE ALL-PERVASIVENESS OF GAṆITA

"All activities which relate to worldly, Vedic or religious affairs make use of enumeration (Saṅkhyāna). In the art of love, economics, music, dramatics, in the art of cooking, in medicine, in architecture and such other things, in prosody, in poetics and poetry, in logic, grammar and such other things, and in relation to all that constitute the peculiar value of the arts − the science of calculation (Gaṇita) is held in high esteem. In relation to the movement of the sun and other heavenly bodies, in connection with eclipses and conjunction of planets, and in the determination of direction, position and time (Tripraśna) and in (knowing) the course of the moon – indeed in all these it (Gaṇita) is accepted (as the sole means).

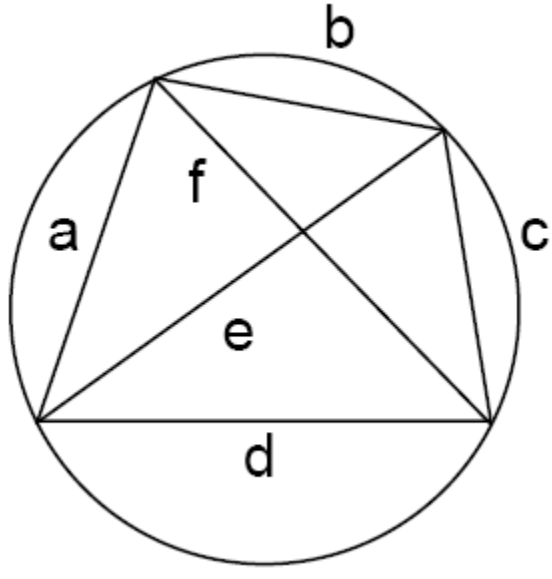
MAHĀVĪRĀCĀRYA ON THE ALL-PERVASIVENESS OF GAṆITA

"The number, the diameter and perimeter of the islands, oceans and mountains; the extensive dimensions of the rows of habitations and halls belonging to the inhabitants of the world, of the interspaces between the worlds, of the world of light, of the world of the gods and of the dwellers in hell, and other miscellaneous measurements of all sorts – all these are understood by the help of Gaṇita. The configuration of living beings therein, the length of their lives, their eight attributes and other similar things, their staying together, etc. – all these are dependent on Gaṇita.

Why keep talking at length? In all the three worlds involving moving and non-moving entities, there is nothing that can be without the science of calculation (Gaṇita). "

[*Gaṇitasārasaṅgraha of* Mahāvīrācārya (c.850), 1.9-16]

BRAHMAGUPTA'S FORMULAE FOR CYCLIC QUADRILATERALS (c.628)



The diagonals *e, f* are given in terms of the sides a, b, c, d, by the formulae

$$e = \sqrt{\frac{(ad+bc)(ac+bd)}{ab+cd}}, \quad f = \sqrt{\frac{(ab+cd)(ac+bd)}{ad+bc}}.$$

The area is given by

$$A = [(s-a)\,(s-b)\,(s-c)\,(s-d)]^{1/2} \text{ with } s = (a+b+c+d)/2$$

BRAHMAGUPTA'S BHĀVANĀ (c.628)

मूलं द्विधेष्टवर्गाद् गुणकगुणादिष्टयुतविहीनाच्च।
आद्यवधो गुणकगुणः सहान्त्यघातेन कृतमन्त्यम्॥
वज्रवधैक्यं प्रथमं प्रक्षेपः क्षेपवधतुल्यः।
प्रक्षेपकशोधकहृते मूले प्रक्षेपके रूपे॥

(ब्राह्मस्फुटसिद्धान्त, कुट्टकाध्याय ६४-६५)

If $x_1^2 - D\, y_1^2 = k_1$ and $x_2^2 - D\, y_2^2 = k_2$ then

$$(x_1 x_2 \pm D\, y_1 y_2)^2 - D\,(x_1 y_2 \pm x_2 y_1)^2 = k_1 k_2$$

In particular given $x^2 - D\, y^2 = k$, we get the rational solution

$$[(x^2 + D\, y^2)/k]^2 - D\,[(2xy)/k]^2 = 1$$

Also, if one solution of the Equation $x^2 - D\, y^2 = 1$ is found, an infinite number of solutions can be found, via $(x, y) \rightarrow (x^2 + D\, y^2, 2xy)$

USE OF BHĀVANĀ WHEN K = -1, ±2, ±4

The Bhāvanā principle can be use to obtain a solution of the equation

$$x^2 - D\,y^2 = 1$$

if we have a solution of the equation

$$x_1^2 - D\,y_1^2 = K \text{ for } K = -1, \pm 2, \pm 4$$

$$\begin{aligned}
K = -1 \ : \ x &= x_1^2 + Dy_1^2, \quad y = 2x_1y_1. \\
K = \pm 2 \ : \ x &= \frac{\left(x_1^2 + Dy_1^2\right)}{2} \quad y = x_1y_1. \\
K = -4 \ : \ x &= \left(x_1^2 + 2\right)\left[\frac{1}{2}\left(x_1^2 + 1\right)\left(x_1^2 + 3\right) - 1\right], \\
y &= \frac{x_1y_1\left(x_1^2 + 1\right)\left(x_1^2 + 3\right)}{2}. \\
K = +4 \ : \ x &= \frac{\left(x_1^2 - 2\right)}{2}, \quad y = \frac{x_1y_1}{2}, \quad \text{if } x_1 \text{ is even }, \\
x &= x_1\frac{\left(x_1^2 - 3\right)}{2}, \ y = \frac{y_1\left(x_1^2 - 1\right)}{2}, \ \text{if } x_1 \text{ is odd.}
\end{aligned}$$

BHĀVANĀ AND RATIONAL APPROXIMATION OF SQUARE-ROOTS

If x and y have been found as a solution of

$x^2 - D\,y^2 = 1$,

then x/y is an approximation to $\sqrt{D}$. Then, by the Bhāvanā

$(x, y) \rightarrow (x^2 + D\,y^2, 2xy)$,

we can generate a series of better and better approximations.

The equation $x^2 - 2y^2 = 1$ has solutions $x = 3$, $y = 2$.

By Bhāvanā, we get another solution $17 = 3^2 + 2.\,2^2$ and $12 = 2.3.2$.

By doing Bhāvanā once again, we get $507 = 17^2 + 2.\,12^2$ and $408 = 2.17.12$.

Thus, $\sqrt{2} \sim 3/2,\ 17/12,\ 509/408 = 1 + 1/3 + 1/(3.4) - 1/(3.4.34)$ (Śulvasūtra approximation).

We can show that this process leads to a series

$$\sqrt{2} \sim 1 + 1/3 + 1/3.4 - 1/3.4.34 - 1/3.4.34.1154 - 1/3.4.34.1154.1331714 - \ldots$$

where $1154 = 34^2 - 2$, $133714 = 1154^2 - 2$, and so on.

CAKRAVĀLA ALGORITHM (c.1050)

To solve $\mathbf{X^2 - D\,Y^2 = 1}$

Set $X_0 = 1$, $Y_0 = 0$, $K_0 = 1$ and $P_0 = 1$.

Given X_i , Y_i , K_i such that $X_i^{\,2} - D\,Y_i^2 = K_i$

First find P_{i+1} so as to satisfy:

(I) $P_i + P_{i+1}$ is divisible by K_i

(II) $|\,P_{i+1}^2 - D\,|$ is minimum.

Then set

$K_{i+1} = (P_{i+1}^{\,2} - D)/K_i$

$Y_{i+1} = (Y_i\,P_{i+1} + X_i)/\,|K_i|$

$X_{i+1} = (X_i\,P_{i+1} + DY_i)/\,|K_i|$

These satisfy $X_{i+1}^{\,2} - D\,Y_{i+1}^{\,2} = K_{i+1}$

Iterate till $K_{i+1} = \pm 1, \pm 2$ or ± 4, and then use Bhāvanā if necessary.

BHĀSKARA'S EXAMPLE: $X^2 - 61\ Y^2 = 1$

I	P_i	K_i	a_i	ε_i	X_i	Y_i
0	0	1	8	1	1	0
1	8	3	5	-1	8	1
2	7	-4	4	1	39	5
3	9	-5	3	-1	164	21

To find P_1: 0+7, 0+8, 0+9 ... divisible by 1. 8^2 closest to 61. $P_1 = 8$, $K_1 = 3$

To find P_2: 8+4, 8+7, 8+10 ... divisible by 3. 7^2 closest to 61. $P_2 = 7$, $K_2 = -4$

After the second step, we have: $39^2 - 61 \times 5^2 = -4$

Since K= -4, we can use Bhāvanā principle to obtain

$X = (39^2 + 2)\ [(½)\ (39^2 + 1)\ (39^2 + 3) - 1] =$ **1,766,319,049**

$Y = (½)\ (39 \times 5)\ (39^2 + 1)\ (39^2 + 3) =$ **226,153,980**

$$1766319049^2 - 61 \times 226153980^2 = 1$$

OPTIMALITY OF THE CAKRAVĀLA ALGORITHM

What is intriguing is that the same example, $X^2 - 61\ Y^2 = 1$, was posed, five hundred years later, as a challenge by the famous French mathematician Pierre de Fermat in February 1657 to his colleagues in France. He later on posed this and other Vargaprakṛti equations $X^2 - D\ Y^2 = 1$ (with different values of D) as a challenge to the British mathematicians.

The British mathematicians Wallace and Brouncker did come up with a method of solution, which was later systematised as an algorithm, based on the so called regular continued fraction development of the square-root of D, by Euler and Lagrange in the 1770s.

In 1929, A. A. Krishnaswamy Ayyangar showed that the Cakravāla algorithm corresponds to a so called semi-regular continued fraction expansion, and is also optimal in the sense that it takes much fewer steps to arrive at the solution than the Euler-Lagrange method.

It is now known that on the average the Euler-Lagrange method takes about 40% more number of steps than the Cakravāla.

TĀTKĀLIKA-GATI: INSTANTANEOUS VELOCITY

In astronomy, in order to determine the true longitude of a planet, a Manda-phala which corresponds to the so called equation of centre is added to the mean longitude. While the mean longitude itself varies uniformly with time, the Manda-phala, in the first approximation, is proportional to the sine of the mean longitude. The velocity of the planet therefore varies continuously with time.

$$\theta_t = \theta_0 - \sin^{-1}\left(\frac{r_0}{R}\sin M\right) \approx \theta_0 - \frac{r_0}{R}\, R\sin M.$$

where the anomaly (Kendra) $M =$ Mean longitude – Longitude of apogee. Hence,

$$\frac{\Delta\theta_t}{\Delta t} = \frac{\Delta\theta_0}{\Delta t} - \frac{r_0}{R}(\cos M)\frac{\Delta M}{\Delta t}$$

The expression for the true velocity (Sphuṭa-manda-gati) in terms of cosine (the derivative of sine) appears for the first time in the *Laghumānasa* of Muñjāla (c.932) and *Mahāsiddhānta* of Āryabhaṭa II (c.950).

कोटिफलघ्नी भुक्तिर्गज्याभक्ता कलादिफलम्॥

The *koṭiphala* multiplied by the [mean] daily motion and divided by the radius gives the minutes of the correction [to the rate of the motion].

TĀTKĀLIKA-GATI: INSTANTANEOUS VELOCITY

In his *Siddhānta-śiromaṇi*, Bhāskara II (c.1150) discusses the notion of instantaneous velocity (*tātkālika-gati*) and contrasts it with the so called true daily rate of motion which is the difference of the true longitudes on successive days. He emphasises that the instantaneous velocity is especially relevant in the case of Moon.

> समीपतिथ्यन्तसमीपचालनं विधोस्तु तत्कालजयैव युज्यते ।
> सुदूरसञ्चालनमाद्यया यतः प्रतिक्षणं सा न समा महत्यतः ॥

> In the case of the Moon, the ending moment of a Tithi which is about to end or the beginning time of a Tithi which is about to begin, are to be computed with the instantaneous rate of motion at the given instant of time. The beginning moment of a Tithi which is far away can be calculated with the earlier [daily] rate of motion. All this is because the Moon's rate of motion is large and varies from moment to moment.

In his commentary, *Vāsanā*, Bhāskara emphasises the above point still further.

> **तात्कालिकया भुक्त्या चन्द्रस्य विशिष्टं प्रयोजनम् ।** तदाह "समीपतिथ्यन्तसमीपचालनम्" इति । यत्कालिकश्चन्द्रस्तस्मात् कालाद्गतोवा गम्यो वा यदासन्नस्तिथ्यन्तस्तदा तात्कालिकया गत्या तिथिसाधनं कर्तुं युज्यते । तथा समीपचालनं च । यदा यदा तु दूरतरस्तिथ्यन्तो दूरचालनं वा चन्द्रस्य तदाद्यया स्थूलया कर्तुं युज्यते । स्थूलकालत्वात् । **यतश्चन्द्रगतिर्महत्वात् प्रतिक्षणम् समा न भवति । अतस्तदर्थमयं विशेषोऽभिहितः ।**
>
> In the case of the Moon, this instantaneous rate of motion is especially useful. ...Because of its largeness, the rate of motion of Moon is not the same every instant. Hence, in the case [of Moon] the special [instantaneous] rate of motion is instructed.

PROBLEM OF PLANETARY LATITUDES

In *Siddhāntaśiromaṇi*, Bhāskara also refers to the problem that there are different procedures for computing the latitudes of the interior and exterior planets. He cites Pṛthūdakasvāmin to say that there is no explanation for this anomaly except for the fact that the results tally with observations.

ननु ज्ञशुक्रयोः शिघ्रोच्चपातयुतिं केन्द्रं कृत्वा यो विक्षेप आनीतः स शीघ्रोच्चस्थान एव भवितुमर्हति। न ग्रहस्थाने। यतो ग्रहोऽन्यत्र वर्तते। अत इदमनुपपन्नमिव प्रतिभाति।तथा च ब्रह्मसिद्धान्तभाष्ये। ज्ञशुक्रयोः शीघ्रोच्चस्थाने यावान् विक्षेपस्तावानेव यत्रतत्रस्यापि ग्रहस्य भवति। अत्रोपलब्धिरेव वासना नान्यत् कारणं वक्तुं शक्यत इति चतुर्वेदेनाप्यनध्यवसायोऽत्र कृतः।

"The latitude that is obtained by using the *śīghrocca* and the node must be the latitude at the location of *śīghrocca* and not at the location of the planet, as the planet is somewhere else. Therefore this seems to be without any justification. However even Caturvedācārya (Pṛthūdakasvāmin) has concluded as follows in his commentary on *Brhmasphuṭasiddhnta*: 'The latitude at the location of the *śīghrocca* of the planets Mercury and Venus, corresponds to the latitude of the planet itself wherever the latter may be. Here the agreement (between the calculated results and observations) is the only justification as we are unable to give any other reason."

Later Classical Period (600-1250)

- Mādhavakara (C.700): *Mādhavanidāna or Rugviniścaya*
- Bhadanta Nāgārjuna (c.800): *Rasavaiśeṣikasūtra, Yogaśataka*
- Vṛnda (c.850): *Vṛndamādhava or Siddhayoga*
- Ṭisaṭa (c.850): *Cikitsākālikā*
- Candraṭa (c.900): *Yogaratnasamuccaya*
- Indu (c.900): Commentaries on *Aṣṭāṅgasaṁgraha* and *Aṣṭāṅgahṛdaya*
- Cakrapāṇidatta (c.1050): *Āyurvedadīpikā* on *Carakasaṁhitā. Cakradatta or Cikitsāsaṁgraha. Dravyaguṇasaṁgraha*
- Bhoja (c.1050): *Rajamārtāṇḍa* or *Yogasārasaṁgraha*
- Govindabhagavatpāda (c.1000): *Rasahṛdayatantra*
- Somadeva (c.1100): *Rasendracūḍāmaṇi*
- Vaṅgasena (c.1150): *Cikitsāsārasaṁgraha*
- Aruṇadatta (c.1150): *Sarvāṅgasundarī* on *Aṣṭāṅgahṛdaya*
- Dalhaṇa (c.1200): *Nibandhasangraha* on *Suśrutasaṁhitā*
- Yasodharabhaṭṭa (c.1200): *Rasaprakāśasudhākara*
- Vāgbhaṭa (c.1200): *Rasaratnasamuccaya*
- Śoḍhala (c.1200): *Gadanigraha. Śoḍhalanighaṇṭu*
- Hemādri (c.1250): *Āyurvedarasāyana* on *Aṣṭāṅgahṛdaya*

AL ANDALUSI ON SCIENCE IN INDIA (c. 1068)

"The first nation [to have cultivated science] is India. This is a powerful nation having a large population and a rich kingdom. India is known for the wisdom of its people. Over many centuries, all the kings of the past have recognised the ability of the Indians in all the branches of knowledge. The kings of China ... referred to the king of India as the 'King of Wisdom' because of the Indians' careful treatment of the *ulūm* [sciences] and their advancement in all the branches of knowledge....

To their credit, the Indians have made great strides in the study of numbers and of geometry. They have acquired immense information and reached the zenith of their knowledge in the knowledge of the movement of the stars [astronomy] and the secrets of the skies [astrology] as well as other mathematical studies. After all that, they have surpassed all the other peoples in their knowledge of medical science and the strength of various drugs, the characteristics of compounds, and the peculiarities of substances."

[Said Al Andalusi, *Book of the Categories of Nations*, Tr. S. I. Salim and Alok Kumar, University of Texas Press, 1991, pp. 11-12. Al Andalusi (1029-1070), was an astronomer, philosopher of science and historian, who worked in Toledo.]

AL BIRUNI ON THE STATE OF SCIENCE IN NORTH INDIA (c.1030)

"Now in the following times [after Muhammad ibn Kasim (695-715)] no Muslim conqueror passed beyond the frontier of Kabul and the river Sindh until the days of the Turks when they seized the power in Ghazna. ... Yaminaddaula Mahmud marched into India during a period of thirty years and more. ...

Mahmud utterly ruined the prosperity of the country, and performed there wonderful exploits, by which the Hindus became like atoms of dust scattered in all directions, and like a tale old in the mouth of the people. ...

This is the reason too why the Hindu sciences have retired far away from those parts of the country conquered by us, and have fled to places which our hands cannot yet reach, to Kashmir, Benares and other places."

Al Biruni's *Indica*, Tr. E. C. Sachau, London 1910, Vol I, p.22

INDIAN SCIENCE IN THE MEDIEVAL PERIOD (1250-1750)

Medieval Period (c. 1250-1750)

- Rāmacandra (c.1350): *Prakriyākaumudī*
- Nārāyaṇa Bhaṭṭātiri (c.1600): *Prakriyāsarvasva*
- Bhaṭṭoji Dīkṣita (c.1625): *Siddhāntakaumudī, Prauḍhamanoramā, Śabdakaustubha*
- Kauṇḍabhaṭṭa (c.1650): *Vaiyākaraṇabhūṣaṇa*
- Varadarāja (c.1650): *Laghu-siddhāntakaumudī*, *Sāra-siddhāntakaumudī*
- Nāgeśabhaṭṭa (c.1700): *Mahābhāṣya-pradipodyota, Bṛhacchabdendu-śekhara,Vaiyākaraṇa-siddhāntamañjūṣā, Paramalaghumañjūṣā*, *Paribhāṣenduśekhara,* etc.

Grammars of Other Languages

- Telugu: *Triliṅga-śabdānuśāsana* (c.1300)
- Persian: *Pārasīprakāśa* (c.1575)
- Kannada: *Karnāṭaka-śabdānuśāsana* (c.1600)

Medieval Period (1250 – 1750)

- *Vākyakaraṇa* (c. 1300): Basic text of *Vākya* system which computes true longitudes directly for a suitable cycle of days.
- *Gaṇitasārakaumudī* (in Prākṛta) of Ṭhakkura Pherū (c.1300) and other works in regional languages such as *Vyavahāragaṇita* (Kannaḍa) of Rājāditya, *Pāvulūrigaṇitamu* of Pāvulūri Mallana in Telugu.
- *Gaṇitakaumudī* and *Bījagaṇitāvataṃsa* of Nārāyaṇa Paṇḍita (c. 1350): The most comprehensive canonical texts on Pāṭīgaṇita and Bījagaṇita.
- *Yantrarāja* of Mahendrasūri (c.1370)
- *Makaranda-sāriṇī* (c.1478)
- Works of Jñānarāja (c.1500), Gaṇeśa Daivajña (b.1507), Sūryadāsa (c.1541) and Kṛṣṇa Daivajña (c.1600): Commentaries with Upapattis
- Works of Nityānanda (c.1639), Munīśvara (c.1646) and Kamalākara (c.1658): Engaging with Islamic Astronomical Tradition.
- Mathematics and Astronomy in the Court of Savai Jayasiṁha (1688-1743): The five observatories. Translation from Persian of Euclid and Ptolemy.

DEVELOPMENT OF INDIAN ASTRONOMY & MATHEMATICS

Medieval Period (1250 – 1750)

- Mādhava (c.1350): Founder of the Kerala School. Infinite series for **π**, sine and cosine functions and fast convergent approximations to them. *Veṇvāroha*, *Agaṇita* method, and other contributions.
- Parameśvara (c.1380-1460): *Dṛggaṇita*, *Goladīpikā*, Commentaries on *Āryabhaṭīya, Sūryasiddhānta,* etc. Over 55 years of observations.
- Nīlakaṇṭha Somayājī (c.1444-1540): *Tantrasaṅgraha*, *Aryabhatiyabhasya, Siddhantadarpana, Jyotirmīmāṁsā, Grahasphutnayane Viksepavasana,* etc. Revised planetary model.
- Systematic exposition of Mathematics and Astronomy with proofs in *Yuktibhāṣā* (in Malayalam) of Jyeṣṭhadeva (c.1530) and commentaries *Kriyākrmakarī* and *Yuktidīpikā* of Śaṅkara Vāriyar (c.1540).
- Acyuta Piṣāraṭi (c.1550-1621): *Sphuṭanirṇayatantra*.
- Putumana Somayājī (Post 1550): *Karaṇapaddhati*. Improved Methods for devising Vākyas.

MĀDHAVA SERIES FOR π AND END-CORRECTION TERMS

The following verses of Mādhava are cited in *Yuktibhāṣā* and *Kriyākramarī*, which also present a detailed derivation of the relation between the diameter and circumference of a circle:

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते।
त्रिशरादिविषमसङ्ख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात्॥१॥

यत्सङ्ख्याऽत्र हरणे कृते निवृत्ता हृतिस्तु जामितया।
तस्या ऊर्ध्वगताया समसङ्ख्या तद्दलं गुणोऽन्ते स्यात्॥२॥

तद्वर्गो रूपयुतो हारो व्यासाब्धिघाततः प्राग्वत्।
ताभ्यामाप्तं स्वमृणे कृते धने क्षेप एव करणीयः॥३॥
लब्धः परिधिः सूक्ष्मो बहुकृत्वो हरणतोऽतिसूक्ष्मः स्यात्॥४॥

The first verse gives the Mādhava series (Leibniz series)

$$Paridhi = 4 \times Vy\bar{a}sa \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\right)$$

MĀDHAVA SERIES FOR π AND END-CORRECTION TERMS

The Mādhava series for the circumference of a circle (in terms of odd numbers p = 1, 3, 5, ...) can be written in the form

$$C = 4d\,[1 - 1/3 + \ldots + (-1)^{(p-1)/2}\, 1/p + \ldots]$$

This is an extremely slowly convergent series. In order to facilitate computation, Mādhava has given a procedure of using end-correction terms (Antya-saṃskāra), of the form

$$C = 4d\left[1 - \frac{1}{3} + \ldots + (-1)^{\frac{(p-1)}{2}}\frac{1}{p} + (-1)^{\frac{(p+1)}{2}}\frac{1}{a_p}\right]$$

In fact, the famous verses of Mādhava, which give the relation between the circumference and diameter, also include the end-correction term

$$C = 4d\,[1 - 1/3 + \ldots + \ldots + (-1)^{(p-1)/2}\, 1/p$$
$$+ (-1)^{(p+1)/2}\, \{(p+1)/2\}/\{(p+1)^2 + 1\}]$$

MĀDHAVA SERIES FOR π AND END-CORRECTION TERMS

Mādhava has also given a finer end-correction term

अन्ते समसङ्ख्यादलवर्गः सैको गुणः स एव पुनः ॥
युगगुणितो रूपयुतः समसङ्ख्यादलहतो भवेद् हारः ।

$$C = 4d\,[1 - 1/3 + \ldots + \ldots + (-1)^{(p-1)/2}\, 1/p$$
$$+(-1)^{(p+1)/2}\,[\{(p+1)/2\}^2 + 1]/[\{(p+1)^2 + 5\}\{(p+1)/2\}]$$

To Mādhava is attributed a value of π accurate to eleven decimal places which is obtained by just computing fifty terms with the above correction.

विबुधनेत्रगजाहिहुताशनत्रिगुणवेदभवारणबाहवः ।
नवनिखर्वमिते वृतिविस्तरे परिधिमानमिदं जगदुर्बुधाः ॥

The π value given above is:

$$\pi \approx \frac{2827433388233}{9 \times 10^{11}} = 3.141592653592\ldots$$

A HISTORY OF APPROXIMATIONS TO π

	Approximation to π	Accuracy (Decimal places)	Method Adopted
Rhind Papyrus - Egypt (Prior to 2000 BCE)	256/81 = 3.1604	1	Geometrical
Babylon (2000 BCE)	25/8 = 3.125	1	Geometrical
Śulva Sūtras (Prior to 800 BCE)	3.0883	1	Geometrical
Jaina Texts (500 BCE)	√ (10) = 3.1623	1	Geometrical
Archimedes (250 BCE)	3 10/71 < π < 3 1/7	2	Polygon doubling (6.2^4 = 96 sides)
Ptolemy (150 CE)	3 17/120 = 3. 141666	3	Polygon doubling (6.2^6 = 384 sides)
Lui Hui (263)	3.14159	5	Polygon doubling (6.2^9 = 3072 sides)
Tsu Chhung-Chih (480?)	355/113 = 3.1415929 3.1415927	6 7	Polygon doubling (6.2^9 = 12288 sides)
Āryabhaṭa (499)	62832/20000 = 3.1416	4	Polygon doubling (4.2^8 = 1024 sides)

A HISTORY OF APPROXIMATIONS TO π

	Approximation to π	Accuracy (Decimal places)	Method Adopted
Mādhava (1375)	$2827433388233/9.10^{11}$ = 3.141592653592	11	Infinite series with end corrections
Al Kashi (1430)	3.1415926535897932	16	Polygon doubling (6.2^{27} sides)
Francois Viete (1579)	3.1415926536	9	Polygon doubling ($6.\ 2^{16}$ sides)
Romanus (1593)	3.1415926535.....	15	Polygon doubling
Ludolph Van Ceulen (1615)	3.1415926535.....	32	Polygon doubling (2^{62} sides)
Wildebrod Snell (1621)	3.1415926535.....	34	Modified polygon doubling (2^{30} sides)
Grienberger (1630)	3.1415926535.....	39	Modified polygon doubling
Isaac Newton (1665)	3.1415926535.....	15	Infinite series
Abraham Sharp (1699)	3.1415926535.....	71	Infinite series for $\tan^{-1}(1/\sqrt{3})$
John Machin (1706)	3.1415926535.....	100	Infinite series relation $\pi/4 = 4\tan^{-1}(1/5) - \tan^{-1}(1/239)$
Ramanujan (1910, 1914), Gosper (1985)		17 Million	Modular Equation
Kondo, Yee (2010)		5 Trillion	Modular Equation

A HISTORY OF EXACT RESULTS FOR π

Mādhava (1375)	$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$ $\pi/\sqrt{12} = 1 - 1/3.3 + 1/3^2.5 - 1/3^3.7 + \ldots$ $\pi/4 = 3/4 + 1/(3^3-3) - 1/(5^3-5) + 1/(7^3-7) - \ldots$ $\pi/16 = 1/(1^5+4.1) - 1/(3^5+4.3) + 1/(5^5+4.5) - \ldots$
Francois Viete (1593)	$2/\pi = \sqrt{[1/2]}\,\sqrt{[1/2 + 1/2\sqrt{(1/2)}]}\,\sqrt{[1/2 + 1/2\sqrt{(1/2+1/2\sqrt{(1/2)})}]}\ldots$ (Infinite product)
John Wallis (1655)	$4/\pi = (3/2)(3/4)(5/4)(5/6)(7/6)(7/8)\ldots$ (Infinite product)
William Brouncker (1658)	$4/\pi = 1+ 1^2/2+ 3^2/2+ 5^2/2+ \ldots$ (Continued fraction)
Isaac Newton (1665)	$\pi = 3\sqrt{3}/4 + 24\,[1/3.8 - 1/5.32 - 1/7.128 - 1/9.512 - \ldots]$
James Gregory (1671)	$\tan^{-1}(x) = 1 - x/3 + x^2/5 - \ldots$
Gottfried Leibniz (1674)	$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$
Abraham Sharp (1699)	$\pi/\sqrt{12} = 1 - 1/3.3 + 1/3^2.5 - 1/3^3.7 + \ldots$
John Machin (1706)	$\pi/4 = 4\tan^{-1}(1/5) - \tan^{-1}(1/239)$

Ramanujan (1910, 1914)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!\,(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

REVISED PLANETARY MODEL OF NĪLAKAṆṬHA SOMAYĀJĪ

In his celebrated work *Tantrasaṅgraha* (c.1500), Nīlakaṇṭha Somayājī came up with a fundamental revision of the traditional planetary model, by proposing that, in the case of Mercury and Venus, what were till then thought of as Śīghroccas should be understood as the mean planets themselves, and that the equation of centre should be applied to them. He also proposed that the latitudes of these planets should be calculated from this Manda-corrected mean planet in the same way as for other planets.

In his *Āryabhaṭīyabhāṣya*, Nīlakaṇṭha explains in detail the rationale for his revised model. He shows that it provides a coherent account of the latitudinal motion of the interior planets, based on the understanding that the motion in latitude is of the planet itself and not of some Śīghrocca.

शीघ्रवशाच्च विक्षेप उक्तः। कथमेतद्युज्यते। ननु स्वबिम्बस्य विक्षेपः स्वभ्रमणवशादेव भवितुमर्हति। न पुनरन्यभ्रमणवशादिति। सत्यम्। न पुनरन्यस्य भ्रमणवशादन्यस्य विक्षेप उपपद्यते। तस्मात् बुधोऽष्टाशीत्यैव दिनैः स्वभ्रमणवृत्तं पूरयति।... एतच्च नोपपद्यते यदेकेनैव संवत्सरेण तत्परिभ्रमणमुपलभ्यते नैवाष्टाशीत्या दिनैः। सत्यम् भगोलपरिभ्रमणं तस्यप्येकेनैवाब्देन।

The latitudinal motion is said to be due to that of the Śīghrocca. How is this appropriate? Isn't the latitudinal motion of a body dependent on the motion of that body only, and not because of the motion of something else? The latitudinal motion of one body cannot be obtained as being due to the motion of another. Hence [we should conclude that] Mercury goes around its own orbit in 88 days... However this also is not appropriate because we see it going around [the Earth] in one year and not in 88 days. True, the period in which Mercury completes one full revolution around the Bhagola (the celestial sphere) is one year only [like the Sun] ...

एतदुक्तं भवति। **तयोर्भ्रमणवृत्तेन न भूः कबलीक्रियते। ततो बहिरेव सदा भूः।** भगोलैकपार्श्व एव तद्वृत्तस्य परिसमाप्तत्वात् तद्भगणेन न द्वादशराशिषु चारः स्यात्। ... तथाप्यादित्यभ्रमणवशादेव द्वादशराशिषु चारः स्यात्। ... **यथा कुजादीनामपि शीघ्रोच्चं स्वमन्दकक्ष्यामण्डलादिकमाकर्षति एवमेतयोरपि। अनयोः पुनस्तदाकर्षण-वशादेव द्वादशराशिषु चारः इति।**

All this can be explained thus: **Their [Mercury and Venus] orbits do not circumscribe the earth. The Earth is always outside their orbit.** Since their orbit is always confined to one side of the [geocentric] celestial sphere, in completing one revolution they do not go around the twelve signs (Rāśis)... It is only due to the revolution of the Sun [around the Earth] that they (i.e. the interior planets, Mercury and Venus) complete their movement around the twelve signs [and complete their revolution of the Earth]... **Just as in the case of the Jupiter etc. [exterior planets] the Śīghrocca (i.e., the mean Sun) attracts [and drags around] the Manda-orbits on which they move, in the same way it does for these [interior] planets also. And it is due to this attraction that these [interior planets] move around the twelve signs.**

The above passage exhibits the clinching argument employed by Nīlakaṇṭha. From the fact that the motion of the interior planets was characterized by two different periods, one for their latitudinal motion and another for their motion in longitude, Nīlakaṇṭha arrived at his revolutionary discovery concerning the motion of the interior planets: That they go around the Sun in orbits that do not circumscribe the Earth in a period that corresponds to the period of their latitudinal motion; they go around the Zodiac in one year being dragged around the Earth by the Sun.

It was indeed well known to the ancients that the exterior planets, Mars, Jupiter and Saturn, go around the Earth and that they also go around the Sun in the same mean period, because their geocentric orbit was outside that of the Sun. Nīlakaṇṭha was the first savant in the history of astronomy to clearly derive from the computational scheme, and not from any speculative or cosmological argument, that the interior planets go around the Sun and the period of their motion around Sun is also the period of their latitudinal motion.

HISTORY OF PLANETARY MODELS

Indian Tradition	Greco-European Tradition	Islamic Tradition
Vedāṅga Jyotiṣa	Babylonian Tables	
Siddhāntas	Eudoxus (380 BCE)	
	Aristotle (350)	
	Apollonius (230)	
	Hipparchus (150 BCE)	
	Ptolemy (150 CE)	
Āryabhaṭa (499 CE)		
Varāhamihira (550)		
Brahmagupta (628)		
Bhāskara I (630)		
Vaṭeśvara (906)		
Muñjāla (930)		
Bhāskara II (1150)		Al Haytham (1000)
		Al Urdi , Al Tusi & Al Shirazi (1250-75)
Mādhava (1380)		Al Shatir (1350)
Parameśvara (1430)		
Nīlakaṇṭha (1500)		
Jyeṣṭhadeva (1530)		
Acyuta (1575)	Copernicus (1543)	
	Tycho Brahe (1587)	
	Kepler (1609)	

The traditional Indian planetary model had a better formulation of the latitudinal motion in terms of the Śighrocca.

This is what enabled Nīlakaṇṭha to arrive at the correct rule for applying the equation of centre for an interior planet to the mean heliocentric planet (as opposed to the mean Sun), and develop a satisfactory theory of latitudes for the interior planets. Such a theory came up in the Greco-European astronomical tradition only hundred years later in the work of Kepler (c.1609).

It should be noted that the models proposed by Copernicus (c.1543) and Tycho Brahe (c.1583) retined the Ptolemaic formulation of the equation of centre for interior planets as also the extremely cumbersome latitude theory of Ptolemy. As Kepler remarked:

> Copernicus, ignorant of his own riches, took it upon himself for the most part to represent Ptolemy, not nature, to which he had nevertheless come the closest of all.

JANTAR MANTAR AT JAIPUR (C.1734)

Medieval Period (1250-1750)

- *Śārṅgadharasaṃhitā* (c.1300)
- Dāmodara (c.1300): *Āyurvedacintamaṇī*
- Gopaladāsa (c.1350): *Cikitsāmṛta*
- Trimallabhaṭṭa (c.1450): *Yogataraṅgiṇī*. *Bṛhadyogataraṅgiṇī*.
- Śivadāsasena (c.1500): *Tattvabodhinī* on *Carakasaṁhitā*. *Tattvabodha* on *Aṣṭāṅgahṛdaya*. *Tattvacandrikā* on *Cikitsāsaṁgraha*.
- Bhāvamiśra (c.1535): *Bhāvaprakāśa* and *Nighaṇṭu*
- Kotthūru Basavarāju (c.1550): *Basavarājīyam* (in Telugu)
- Toḍaramalla (c.1575): *Āyurvedasaukhya*
- Lolimbarāja (c.1600): *Vaidyajīvana*. *Vaidyakakāvya* (in Marathi)
- Kāśīrāmavaidya (c.1625): *Gudharthadīpikā* on *Śārṅgadharasaṁhitā*
- Rudrabhaṭṭa (c.1650): *Ayurvedadīpikā* on *Śārṅgadharasaṁhitā*
- Raghunātha (c.1675): *Bhojanakutūhala*
- *Yogaratnākara* (c.1700)
- Govindadāsa (c.1700): *Bhaiṣajyaratnāvalī*
- Viśrāma (c.1750): *Vyādhinigraha*
- Pratapasimha (c.1750) : *Amṛtasāgara* (in Marwari)

[Several Nighaṇṭu and Rasaśāstra texts were also composed in this period]

INDIAN SCIENCE IN THE MODERN PERIOD (POST 1750)

INDIAN SCIENCE OF LANGUAGES AND DEVELOPMENT OF MODERN LINGUISTICS

"Of particular interest is the stress laid on the ‘small number of primitive elements’, themselves not used (i.e., themselves abstract) from which the Sanskrit grammarians are said to derive ‘the infinite variety of actual forms in use.’ "

[J.F.Staal on Francois Pons’ letter of 1740 (published 1743) in, *A Reader on the Sanskrit Grammarians*, MIT Press, 1972, p.30]

"Without Indian grammarians and phoneticians whom he [William Jones (1746-1794)] introduced and recommended to us, it is difficult to imagine our nineteenth century school of phonetics."

[J.R.Firth, Transactions of Philosophical Society, 1946, p.92]

"The algebraic formulation of Pāṇini's rules was not appreciated by the first Western students; they regarded the work as abstruse or artificial. ... The Western critique was muted and eventually turned into praise when modern schools of linguistics developed sophisticated notation systems of their own. Grammars that derive words and sentences from basic elements by a string of rules are obviously in greater need of symbolic code than paradigmatic or direct method practical grammars....

It is a sad observation that we did not learn more from Pāṇini than we did, that we recognised the value and the spirit of his 'artificial' and 'abstruse' formulations only when we had independently constructed comparable systems. The Indian New Logic (*navya-nyāya*) had the same fate: only after Western mathematicians had developed a formal logic of their own and after this knowledge had reached a few Indologists, did the attitude towards the *navya-nyāya* school change from ridicule to respect. "

H. Scharfe, *Grammatical Literature*, Wiesbaden 1977, p.112, 115.

INDIAN SCIENCE OF LANGUAGE AND MODERN LINGUISTICS

"The Descriptive Grammar of Sanskrit, which Pāṇini brought to its perfection, is one of the greatest monuments of human intelligence and an indispensible model for the description of languages. "

[L. Bloomfield, Review of Liebich, Konkordanz das Pāṇini-Candra, *Language*, 5, 267-276, 1929]

"The idea that a language is based on a system of rules determining the interpretation of its infinitely many sentences is by no means novel. Well over a century ago, it was expressed with reasonable clarity by Wilhelm von Humboldt in his famous but rarely studied introduction to general linguistics (Humboldt 1836). His view that a language 'makes infinite use of finite means' and that a grammar must describe the process that makes this possible... Pāṇini's grammar can be interpreted as a fragment of such a 'generative grammar' in essentially the contemporary sense of this term."

[N. Chomsky, *Aspects of the Theory of Syntax*, MIT Press, 1964, p.v]

INDIAN SCIENCE OF LANGUAGE AND MODERN LINGUISTICS

“Modern linguistics acknowledges it as the most complete generative grammar of any language yet written and continues to adopt technical ideas from it ”.

[P. Kiparsky, Pāṇinian Linguistics, in *Encyclopaedia of Language and Linguistics*, VI, 1994]

“Pāṇini's grammar is universally admired for its insightful analysis of Sanskrit...Generative linguists for their part have marvelled especially at its ingenious technical devices, and at intricate system of conventions governing rule application and rule interaction that it presupposes, which seem to uncannily anticipate ideas of modern linguistic theory (if only because many of them were originally borrowed from Pāṇini in the first place.)...

[P. Kiparsky, On the Architecture of Pāṇini's Grammar, 2002]

AN ACCOUNT OF INDIAN ASTRONOMY (C.1770)

"While waiting in Pondicherry for the Transit of 1769, Le Gentil tried to gather information about native astronomy...

Le Gentil eventually contacted a Tamil who was versed in the astronomical methods of his people. With the help of an interpreter he succeeded in having computed for him the circumstances of the lunar eclipse of 1765 August 30, which he himself had observed and checked against the best tables of his times, the tables of Tobias Mayer [1752].

The Tamil Method gave the duration of the Eclipse 41 second too short, the tables of Mayer 1 minute 8 seconds too long; for the totality the Tamil was 7 minutes 48 seconds too short, Mayer 25 seconds too long.

These results of the Tamil astronomer were even more amazing as they were obtained by computing with shells on the basis of memorised tables and without any aid of theory."

AN ACCOUNT OF INDIAN ASTRONOMY (c.1770)

> "Le Gentil says about these computations: ‘They did their astronomical calculations with swiftness and remarkable ease without pen and pencil; their only accessories were cauries... This method of calculation appears to me to be more advantageous in that it is faster and more expeditious than ours.’"

[Neugebauer, *A History of Ancient Mathematical Astronomy* , Vol. III, Springer , 1975, p.20, (Le Gentil's quote translated from French)]

What Neugebauer is referring to as “Tamil method” is nothing but the Vākya method developed in south India, especially by Kerala Astronomers.

Neugebauer also refers to the report of John Warren (in his *Kālasaṅkalita*) about the calculation of a lunar eclipse in 1825 June 1, where the Tamil method predicted midpoint of the eclipse equally accurately with an error of about 23 minutes.

CONTINUING TRADITION OF INDIAN ASTRONOMY (c.1820)

Śaṅkaravarman (1784-1839): Raja of Kaḍattanāḍ in Malabar. Due to the wars with Hyder and Tipu, he is supposed to have spent his early years with Mahārāja Svāti Tirunāl at Tiruvanantapuram.

In 1819, He wrote *Sadratnamālā* (one of the four works mentioned by in a famous article by Charles Whish in 1835), an Astronomical manual following largely the Parahita system. He also wrote his own Malayalam commentary, perhaps a few years later (published along with the text in Kozhikode in 1899).

Chapter I has interesting algorithms for calculation of square and cube roots. Chapter IV deals with computation of sines.

Śaṅkaravarman also gives the following value of π which is accurate to 17 decimal places: $\pi \approx 3.14159265358979324$

CONTINUING TRADITION OF INDIAN ASTRONOMY (c.1870)

Candraśekhara Sāmanta (1835-1904): Popularly known as Paiṭhāni Sāmanta, he had traditional Sanskrit education. Starting from around 1858, he carried out extensive observations for over eleven years, with his own versatile instruments, with a view to to improve the almanac of Puri Jagannātha Temple.

He wrote his *Siddhāntadarpaṇa* containing nearly 2,500 verses in 1869 (published later at Calcutta in 1899). Based on his observations, Sāmanta improved the parameters of the traditional works, he detected and incorporated all the three major irregularities of lunar motion, and improved the traditional estimates of the Sun-Earth distance.

In Chapter V of his work, Sāmanta has presented his planetary model where all the planets move around th Sun, which moves around the Earth.

ACCOUNTS OF INDIAN MEDICINE IN EIGHTEENTH CENTURY

There is a detailed account of the practise of inoculation against smallpox due to J. Z. Holwell, Governor of Bengal in 1767. According to him, this was done by physicians hailing from a number of "colleges" in Vrindavan, Allahabad, Benares, etc, who visited different places annually for this purpose. On the efficacy of this practice, Holwell says:

> "When the before recited treatment of the inoculated is strictly followed, it is next to a miracle to hear, that one in a million fails of receiving the infection, or of one that miscarries under it...Since, therefore, this practice of the East has been followed without variation, and with uniform success from the remotest times, it is but justice to conclude, it must have originally been founded on rational principles and experiment....They lay it down as a principle that the immediate or the instant cause of the smallpox exists in the mortal part of every human and animal form; that the mediate (or second) acting cause, which stirs up the first...is the multitude of animalculae floating in the atmosphere; that these are the cause of all the epidemical diseases, but more particularly of the smallpox." [cited from Dharampal, *Indian Science and Technology in the Eighteenth Century*, Delhi 1971, pp.153-6]

According to Dharampal, "inoculation against small pox seems to have been universal ... in large parts of Northern and Southern India till it was banned ... from around 1802-3." [ibid. p. *xliv*]

The Indian practise of plastic surgery also received considerable attention. In 1794, the *Gentleman's Magazine* published an account of such a surgery done to fix the nose of a solder by a kumbhar (potter) near Pune. The same year, Helenus Scott, a British medical officer in that area, sent a sample of the "cement" that was used in the operation to Sir Joseph Banks, the President of Royal Society (Dharampal, ibid., pp.270-1). A more detailed account was published by J. C. Corpue in 1816. He noted:

> "I did myself the honour to write to Sir Charles Mallet, who had resided many years in India and who obligingly confirmed to me the report, that this had been a common operation in India, from time immemorial; adding, that it had always been performed by the caste of potters, or brick-makers, and that though not invariably, it was usually successful." [J. C. Corpue, *An Account of Two Successful Operations for Restoring a Lost Nose*, London 1816, p.39.]

SOME IMPORTANT ĀYURVEDA TEXTS AND COMMENTARIES (1750-1900)

- Nārāyaṇadāsa (c.1760): Revised *Rājavallabhanighaṇṭu*
- Gangādhara Kavirāja (c.1799-1885): *Vivṛti* on *Rājavallabhanighaṇṭu*
- Dattārāma (c.1800): *Bṛhannighaṇṭuratnākara*
- Vishnu Vasudeva Godavole: *Nighaṇṭuratnākara* (1867)
- Udoy Chund Dutt : *The Materia Medica of the Hindus* (1877)
- Umesh Chandra Gupta: *Vaidyakaśabdasindhu* (1888)
- Gangādhara Kavirāja (c.1799-1885): *Jalpakalpataru* on *Carakasaṁhitā* (1868)
- Vinodalala Sengupta: *Āyurvedavijñāna* (1887)
- Devendranatha Sengupta & Upendranatha Sengupta: *Āyurveda-saṁgraha* (Bengali, 1892)
- Katobhaṭṭa: *Nighaṇṭusaṁgraha* (1893)
- Śāligramavaidya: *Śāligramanighaṇṭu* (1896)
- Krishna Ram Bhat: *Siddhabheṣajamaṇimālā* (1896)

MAHATMA GANDHI ON INDIAN INDIGENOUS EDUCATION IN THE 19TH CENTURY (1931)

"We have the education of this future State. I say without fear of my figures being challenged successfully, that today India is more illiterate than it was fifty or a hundred years ago, and so is Burma, because the British administrators, when they came to India, instead of taking hold of things as they were, began to root them out. They scratched the soil and began to look at the root, and left the root like that, and the beautiful tree perished. The village schools were not good enough for the British administrator, so he came out with his programme. …

I defy anybody to fulfil a programme of compulsory primary education of these masses inside of a century. This very poor country of mine is ill able to sustain such an expensive method of education. Our State would revive the old village schoolmaster and dot every village with a school both for boys and girls….

We give this medical aid, not through the very expensive methods that the Western doctors teach us, but we revive our own ancient treatment. Every village once had its own medical man. ..."

Mahatma Gandhi, Speech at Chatham House, London, October 30, 1931

REPORTS ON INDIAN INDIGENOUS EDUCATION SYSTEM IN EARLY 19TH CENTURY

"If a good system of agriculture, unrivalled manufacturing skill, a capacity to produce whatever can contribute to convenience or luxury; schools established in every village, for teaching reading, writing, and arithmetic; the general practice of hospitality and charity among each other; and above all a treatment of the female sex, full of confidence, respect and delicacy, are among the signs which denote a civilised people, then the Hindus are not inferior to the nations of Europe; and if civilisation is to become an article of trade between the two countries, I am convinced that this country [England] will gain by the import cargo."

[Thomas Munro's Testimony before a Committee of House of Commons April 12, 1813]

"We refer with particular satisfaction upon this occasion to that distinguished feature of internal polity which prevails in some parts of India, and by which the instruction of the people is provided for by a certain charge upon the produce of the soil, and other endowments in favour of the village teachers, who are thereby rendered public servants of the community." [Public Despatch from London to Bengal, June 3, 1814]

REPORTS ON INDIGENOUS EDUCATION IN 19TH CENTURY

"There are probably as great a proportion of persons in India who can read, write and keep simple accounts as are to be found in European countries…" [Annual Report of Bombay Education Society 1819]

"I need hardly mention what every member of the Board knows as well as I do, that there is hardly a village, great or small, throughout our territories, in which there is not at least one school, and in larger villages more; many in every town, and in large cities in every division; where young natives are taught reading, writing and arithmetic, upon a system so economical, from a handful or two of grain, to perhaps a rupee per month to the school master, according to the ability of the parents, and at the same time so simple and effectual, that there is hardly a cultivator or petty dealer who is not competent to keep his own accounts with a degree of accuracy, in my opinion, beyond what we meet with amongst the lower orders in our own country; whilst the more splendid dealers and bankers keep their books with a degree of ease, conciseness, and clearness I rather think fully equal to those of any British merchants."

Minute of G. Prendargast, Member Bombay Governor's Council, April 1821

INDIGENOUS EDUCATION IN MADRAS PRESIDENCY (c.1825)

The British Government conducted a detailed survey of the indigenous system of education covering all the Districts of the Madras Presidency during 1822-25. The Survey found 11,575 schools and 1094 “colleges” in the Presidency. Summarising the survey information the then Governor Thomas Munro wrote in his Minute of March 10, 1826:

> "It is remarked by the Board of Revenue, that of a population of 12½ millions, there are only 188,000, or 1 in 67 receiving education. This is true of the whole population, but not as regards the male part of it, of which the proportion educated is much greater than is here estimated… if we reckon the male population between the ages of five and ten years, which is the period which boys in general remain at school, at one-ninth … the number actually attending the schools [and colleges] is only 184,110, or little more than one-fourth of that number. ... I am, however, inclined to estimate the portion of the male population who receive school education to be nearer to one-third than one-fourth of the whole, because we have no returns from the provinces of the numbers taught at home...."

INDIGENOUS EDUCATION IN MADRAS PRESIDENCY (c.1825)

Community Profile of Boys Attending School

District	Brahmin	Kshatriya	Vaisya	Sudra	Other Castes	Muslims	Total	Total Population
Telugu Districts	13,893	121	7,676	10,076	4,755	1,639	38,160	4,029,408
% Total	*36.41*	*0.32*	*20.12*	*26.40*	*12.46*	*4.30*		
Malabar	2,230		84	3,697	2,756	3,196	11,963	907,575
% of Total	*18.64*		*0.70*	*30.90*	*23.04*	*26.72*		
Tamil Districts	11,557	369	4,442	57,873	13,196	5,453	92,890	6,622,474
% of Total	*12.44*	*0.40*	*4.78*	*62.30*	*14.21*	*5.87*		
TOTAL	29,721	490	13,449	75,943	22,925	10,644	153,172	12,850,941
% of Total	***19.40***	***0.32***	***8.78***	***49.58***	***14.97***	***6.95***		

Source:Dharampal, *The Beautiful Tree*, Impex India, Delhi 1983, pp.21-22.

INDIGENOUS EDUCATION IN MADRAS PRESIDENCY (c.1825)

The languages of instruction in most of the 11,575 schools were the regional languages. The average period of instruction was around 5-7 years. The subjects taught were reading writing and arithmetic.

The instruction in most of the 1,094 "colleges" or institutions of higher learning was in Sanskrit. Details of the subjects taught are available for the 618 colleges in four districts: 418 taught "Vedam", 198 "Law", 34 Astronomy and Gaṇita and 8 taught "Āndhra Śāstram".

Further, in Malabar, 1594 scholars were receiving higher instruction privately, of whom 808 studied Astronomy (of whom 96 were Dvijas) and 154 Medicine (of whom 31 were Dvijas).

As regards the financial support received by the indigenous schools and colleges the situation was clearly stated by the Collector of Bellary:

> "Of the 533 institutions for education, now existing in this district, I am ashamed to say not one now derives any support from the state... There is no doubt that in former times, especially under the Hindoo Governments very large grants, both in money, and in land, were issued for the support of learning."

INDIGENOUS EDUCATION IN BENGAL PRESIDENCY (c.1835)

William Adam's survey (1835) of indigenous education in selected districts of Bengal and Bihar showed the following interesting subject-wise distribution of institutions of higher learning.

Institutions of Sanscritic learning In some districts of Bengal & Bihar

	Murshidabad	Beerbhoom	Burdwan	South-Bihar	Tirhoot	Total
Number of Institutions	24	56	190	27	56	353
Number of Students (Subject wise)						
Grammar	23	274	644	356	127	1,424
Logic	52	27	277	6	16	378
Law	64	24	238	2	8	336
Literature	2	8	90	16	4	120
Mythology	8	8	43	22	1	82
Astrology	—	5	7	13	53	78
Lexicology	4	2	31	8	3	48
Rhetoric	—	9	8	2	—	19
Medicine	—	1	15	2	—	18
Vedum	—	3	3	5	2	13
Tantra	—	1	2	2	—	5
Mimansa	—	—	—	2	—	2
Sankhya	—	—	—	1	—	1
Total Number of Students	153	362	1,358	437	214	2,524

Adam's survey also showed that textbooks used in these institutions of higher learning included, apart from the ancient canonical texts of the various disciplines, many of the important advanced treatises commentaries and monographs composed during the late medieval period.

These included the works of Bhaṭṭoji Dīkṣita (1625), Kauṇḍabhaṭṭa (c.1650), Hari Dīkṣita and Nāgeśa Bhaṭṭa (c.1700) in Vyākaraṇa, the works of Raghunātha (c.1500), Mathurānātha (c.1570), Viśwanātha (c.1650), Jagadīśa (c.1650) and Gadādhara (c.1650) in Navya-nyāya, the works of Raghunandana (c.1550) in Dharmaśāstra and the works *Vedāntasāra* (c.1450) and *Vedāntaparibhāṣā* (c.1650) in Vedānta.

The period of study in these institutions of higher learning was between ten and twenty-five years. In many of these centres of higher learning a large part of the students came from outside, many from even different regions of India. All the students were taught gratis and outside students were provided in addition free food and lodging.

THE UNIVERSITY OF NAVADVĪPA

On visiting Navadvīpa or Nuddeah in 1787, William Jones wrote to Earl of Spencer that "This is the third University of which I am a member".

An account of Navadvīpa in Calcutta Monthly Register (1791):

> "The grandeur of the foundation of the Nuddeah University is generally acknowledged. It consists of three colleges Nuddeah, Santipore and Gopulparra. Each is endowed with lands for maintaining masters in every science….in the college of Nuddeah alone, there are at present about eleven hundred students and one hundred and fifty masters. Their numbers, it is true, fall very short of those in former days. **In Rajah Roodre's time (circa 1680) there were at Nuddaeah no less than four thousand students** and masters in proportion. The students that come from distant parts are generally of a maturity in years, ... yet they say, to become a real Pundit, a man ought to spend twenty years at Nuddeah."

According to Adam, in 1829 there were reported to be 25 schools of learning in Navadvīpa with 500 to 600 students. Some of these schools were still supported by a small allowance from the British Government.

ORIENTALIST-ANGLICIST DEBATE

While the vast indigenous system of school education received no aid or support from the British Government, there were a few centres of higher learning which received some grants, though it was on a much reduced scale from what prevailed prior to British rule.

For instance, the famous Dakshina Fund of the Peshwas which amounted to several lakhs of Rupees distributed each year in the period prior to 1818 was reduced to 35,000 Rupees annually by 1824 after the British takeover.

In the history of modern Indian education, the so called Orientalist-Anglicist debate of the 1830s has often been misrepresented by portraying the Orientalists as great admirers of indigenous learning.

In fact, the Orientalists held the same view as the Anglicists that the indigenous learning was erroneous and outmoded; they only argued that the study of English and “true science” is best engrafted upon the course of education which was esteemed by the Indian people.

MACAULAY'S PRONOUNCEMENT

But it was Macaulay's imperious dismissal all indigenous learning, clearly formulated in his Minute of February 2, 1835, which carried the day in the formulation of the British policy on Indian education:

> " The question now before us is simply whether when it is in our power to teach this language, we shall teach languages in which, by universal confession, there are no books on any subject which deserve to be compared to our own, whether, when we can teach European science, we shall teach systems which, by universal confession, wherever they differ from those of Europe differ for the worse, and whether when we can patronize sound Philosophy and true history, we shall countenance at the public expense medical doctrines which would disgrace an English farrier, astronomy which would move laughter in girls at an English boarding school, history abounding with kings thirty feet high and reigns thirty thousand years long and geography made of seas of treacle and seas of butter."

MACAULAY'S PRONOUNCEMENT

"I think that ... we are free to employ our funds as we chuse; that we ought to employ them in teaching what is best worth knowing, that English is better worth knowing than Sanscrit or Arabic; that the natives are desirous to be taught English, and are not desirous to be taught Sanscrit or Arabic that neither as the languages of law, nor as the languages of religion, have Sanscrit and Arabic any peculiar claim to our encouragement; that it is possible to make natives of this country thoroughly good English scholars, and that to this end our efforts ought to be directed.

In one point I fully agree with the Gentlemen to whose general views I am opposed. I feel with them it is impossible for us, with our limited means, to attempt to educate the body of the people. We must at present do our best to form a class who may be interpreters between us and the millions whom we govern a class of persons Indian in blood and colour, but English in tastes, in opinions, in morals and in intellect. To that class we may leave it to refine the vernacular dialects of this Country, to enrich those dialects with terms of science borrowed from the western nomenclature, and to render them by degrees fit vehicles for conveying knowledge to the great mass of population."

THE GENIUS OF SRINIVASA RAMANUJAN (1887-1920)

In a recent article commemorating the 125th birth-day of Ramanujan, Bruce Berndt has presented the following overall assessment of the results contained in his Notebooks (which record his work prior to leaving for England in 1914):

> "Altogether, the notebooks contain over three thousand claims, almost all without proof. Hardy surmised that over two-thirds of these results were rediscoveries. This estimate is much too high; on the contrary, at least two-thirds of Ramanujan's claims were new at the time that he wrote them, and two-thirds more likely should be replaced by a larger fraction. Almost all the results are correct; perhaps no more than five to ten are incorrect. " [B. Berndt, Notices of AMS (2012), p.1533]

The manuscript of Ramanujan discovered in the Trinity College Library (amongst Watson papers) by G.E. Andrews in 1976, is generally referred as Ramanujan's "Lost Notebook". This seems to pertain to work done by Ramanujan during 1919-20 in India. This manuscript of about 100 pages with 138 sides of writing has around 600 results. G. E. Andrews and B. Berndt have embarked on a four volume edition of all this material in four volumes.

THE ENIGMA OF RAMANUJAN'S MATHEMATICS

For the past hundred years, the problem in comprehending and assessing Ramanujan's mathematics and his genius has centred around the issue of "proof". In 1913, Hardy wrote to Ramanujan asking for proofs of his results. Ramanujan responded by asserting that he had a systematic method for deriving all his results, but that could not be communicated in letters.

Ramanujan's published work in India, and a few of the results contained in the note books have proofs, but they were often said to be sketchy, not rigorous or incomplete. Though, Ramanujan had no doubts about the validity of his results, he was often willing to wait and supply proofs in the necessary format so that his results could be published. But, all the time, he was furiously discovering more and more interesting results.

The Greco-western tradition of mathematics does almost equate mathematics with proof, so that the process of discovery of mathematical results can only be characterised vaguely as "intuition", "natural genius" etc. Since mathematical truths are believed to be non-empirical, there are no systematic ways of arriving at them except by pure logical reason. There are some philosophers who have argued that this "philosophy of mathematics" is indeed barren: it seems to have little validity when viewed in terms of mathematical practice – either in history or in our times.

RAMANUJAN: NOT A NEWTON BUT A MĀDHAVA

In the Indian mathematical tradition, as is known from the texts of the last two to three millennia, mathematics was not equated with proof. Mathematical results were not perceived as being non-empirical and they could be validated in diverse ways. Proof or logical argumentation to demonstrate the results was important. But proofs were mainly for the purpose of obtaining assent for one's results in the community of mathematicians.

In 1913, Bertrand Russell had jocularly remarked about Hardy and Littlewood having discovered a "second Newton in a Hindu clerk". If parallels are to be drawn, Ramanujan may indeed be compared to the legendary Mādhava.

It is not merely in terms of his philosophy of mathematics that Ramanujan is clearly in continuity with the Indian tradition of mathematics. Even in his extraordinary felicity in handling iterations, infinites series, continued fractions and transformations of them, Ramanujan is indeed a successor, a very worthy one at that, of Mādhava, the founder of the Kerala School and a pioneer in the development of calculus.

THE INDEPENDENCE MOVEMENT AND RESURGENCE OF ĀYURVEDA (1880-1947)

- Establishment of the Dabur Company (1884)
- Establishment of Kerala Āyurveda Samajam College at Shoranur and Ārya Vaidya Śālā at Kottakkal (1902)
- Establishment of Madras Āyurveda Pracāriṇi Sabha (1902)
- Āyurvedic Medical Practitioner's Conference in Bombay (1903)
- Establishment of Venkaṭaramaṇa Āyurveda College, Madras (1905)
- First Akhila Bhārata Āyurveda Vaidya Mahā Sammelana, Nasik (1907)
- Mahatma Gandhi's critique of modern medicine (1909)
- Establishment of Benares Hindu University (1916), Aṣṭāṅga Āyurveda College in Calcutta (1916)
- Establishment of Baidyanath Bhavan (1917), Zandu Pharmaceutical Works (1919)
- Resolution of the Indian National Congress 1920.
- Establishment of Venkaṭeśvara Āyurvedic College, Vijayawada (1922)
- Usman Committee (1921-23)
- Establishment of Govt School of Indian Medicine in Madras in 1925. Became College of Indian Medicine in 1947. [Became College of Allopathic Medicine (Kilpauk Medical College) in 1967]
- Establishment of Āyurvedic College, Jamnagar (1944)

ĀYURVEDA AND THE HEALTHCARE SYSTEM IN INDIA

As of 2013, India has nearly 4 lakh registered Āyurvedic (and Siddha) practitioners and 9.2 lakh registered practitioners of Modern Western Medicine (MWM).

There are nearly 2,500 Āyurvedic hospitals with 25,000 beds and 20,000 hospitals of MWM with 6.3 lakh beds.

There are about 300 Āyurvedic colleges (of which around 60 are run by Government) with an intake of 11,000 (UG) and 1,200 (PG) students and 360 colleges of MWM (of which around 170 are run by the Government) with an intake of 45,000 (UG) and 22,500 (PG).

Annual turnover of Indian herbal medicine industry is Rs.2,300 crores, while the MWM pharmaceuticals have a turnover of Rs. 14,500 crores.

The WHO Report on Traditional Medicine Strategy 2002-2005 noted that for 65% of the Indian population Āyurvedic medicine is "the only available source of health care". Several studies have noted that over 80% of Indian population takes recourse to Āyurvedic treatment.

MEAGRE ALLOCATION OF STATE FUNDS FOR ĀYURVEDA

Out of the total outlay of Rs. 400 crores for health in the first three plans (1951-1974), Āyurveda was allocated around Rs. 14 crores (3.5% of the total). The share of Āyurveda (and other ISM) during the sixth to the tenth plans (1980-2007) has been less than 1.6% and in the 11th plan (2007-12), it became 2.9%. Out of the outlay of Rs.300,000 crores for health in the 12th Plan (2002-17), AYUSH have been allocated Rs.10,000 crores (3.3%) which is less than the allocation made for NACO (Rs.11,400 crores).

The allocations for Āyurveda in the annual health budgets are also equally dismal. In 2013, out of the annual health budget of Rs.36,000 crores, AYUSH were allocated Rs.1,250 crores (3.5%) which is less than the allocation for the single institution AIIMS (Rs.1,340 crores).

Thus even though the Āyurvedic system accounts for more than one-third of the registered medical practitioners, one-third of the students studying medicine, and caters to the needs of 80% of the population, the state patronage and support for it in independent India has been truly dismal.

More than 75% of the Āyurvedic colleges and most of the licensed pharmaceuticals (8,000) are supported by the society rather than the state.

TCM AND HEALTHCARE IN CHINA

Like Āyurveda, Traditional Chinese Medicine (TCM) has also had a hoary tradition of over 2500 years. It also came under cloud during the period of foreign domination. In the 19th century acupuncture was banned, and in 1929 the Kuomintang Government proposed a total ban on TCM.

After 1949, TCM became an important part of the national healthcare system. The Chinese Constitution stipulates that "The state develops medical and health services, promotes modern medicine and traditional Chinese medicine..., all for the protection of the people's health."

In 2008, there were over 2.5 lakh registered TCM practitioners and about 20 lakh registered MWM practitioners. In addition more than one-third of the 9 lakh village doctors practised TCM. There were 3,200 TCM hospitals (16% of all hospitals) and 31,000 Clinics (22% of all the clinics).

The Government expenditure on TCM was around $1.2 Billion, which was nearly 7% of the total expenditure on health. The health insurance schemes include TCM, and among the 300 items on the national essential medicines list more than 100 are proprietary TCM drugs. More than 95% of the MWM hospitals have TCM departments or services.

TCM AND HEALTHCARE IN CHINA

In 2008, there were around 50 TCM universities with over 4 lakh students who constitute about one-third of the students studying medicine in China. TCM also formed more than 5% of the curriculum of the students studying in colleges/universities teaching MWM.

TCM hospitals accounted for 13% of all hospital admissions. More than 20% of the outpatients in all hospitals were treated by TCM practitioners. TCM accounted for over 40% of the therapeutic prescriptions at the primary, grass-roots level.

TCM products constitute more than one-third of the total medicine industry in China ($330 Billion in 2014).

The export of TCM products from China were of the order of $1.6 Billion in 2009 and have risen to $ 4.6 Billion in 2014.

In 2015, Tu You You, a scholar of both Western Pharmacology and TCM, was awarded the Nobel Prize in medicine for discovering the anti-malarial drug Artemisinin from the traditional Chinese remedy, sweet wormwood, discussed in 4th century classics of TCM.

A JAPANESE VIEW ON THE STUDY OF NON-WESTERN SCIENCES

"Japanese have been looking to the West ever since the middle of the Edo period [1603-1868]. This not only holds true with the Western culture in general, but in particular in the fields of science and technology. Certainly the discipline of modern science originated in the seventeenth century in Western countries. Before that, however, perspectives of nature, as well as approaches to it, differed considerably according to place, nationality and time. This fact suggests that the modern-scientific view of, and approach to, nature is neither unique nor absolutely correct, and that there are alternatives as to the direction modern science should take.

We hope that the study of the history of sciences in India, China, and Korea, which have all had a great influence upon the Japanese culture including the indigenous science, will make us consider the past, present, and future of our own culture [and] science and enhance our understanding of neighboring countries. It is with this view in mind that we are studying the history of exact science such as mathematics and astronomy from East-Asian and South-Asian countries."

[Takao Hayashi, Science and Engineering Research Institute, Doshisha University http://engineering.doshisha.ac.jp/english/kenkyu/labo/scie/sc_01/index.html]

INDIAN APPROACH TO SCIENCE

ŚĀSTRAS PRESENT SYSTEMATIC PROCEDURES

Most of the canonical texts on different disciplines (Śāstras) in Indian tradition do not present a series of propositions.

Instead they present a series of rules, which serve to specify and characterize systematic procedures in order to accomplish various ends.

The rules are often formulated in the form of Sūtras or verses (Kārikās). According to *Viṣnudharmottarapurāṇa* (3.5.1):

> अल्पाक्षरमसन्दिग्धं सारवद् विश्वतोमुखम्। अस्तोभमनवद्यञ्च सूत्रं सूत्रविदो विदुः॥
>
> Those who know the Sūtra understand it to be concise, unambiguous, pithy, comprehensive, shorn of irrelevancies and blemishless.

These systematic procedures are generally referred to as Vidhi, Kriyā or Prakriyā, Sādhana, Karma or Parikarma, Karaṇa, Upāya etc in different disciplines. We have the famous injunction of *Bhagavadgītā* (16.23):

यः शास्त्रविधिमुत्सृज्य वर्तते कामकारतः। न स सिद्धिमवाप्नोति न सुखं न परां गतिम्॥

ŚĀSTRAS CLEARLY SPECIFY THE PURPOSE OF THE DISCIPLINE

Each Śāstra clearly specifies the purpose of the discipline which also serves to demarcate the topics that legitimately come under its purview.

For instance, we saw that the grammarians characterised their discipline as Śabdānusaśāna, which enables the derivation of all valid utterances of the language.

The mathematicians characterised their discipline as Gaṇita − developing optimal methods of computations involving numbers, figures, etc.

The astronomers characterised their discipline as Kālavidhānaśāstra, the science of determination of time, place and direction, by the study of the motion of celestial bodies.

The Āyurvedic physicians characterised the goal of their discipline as amelioration of diseases of the sick and protection the healthy by achieving Dhātusāmya.

Such clear characterisation of the goals of each science enabled the Indian scientists to develop the appropriate theoretical frameworks and procedures, which were continuously tested and refined in practice.

PĀṆINI'S AṢṬĀDHYĀYĪ AS PARADIGMATIC TEXT

To understand the methodology of Indian sciences, one has to perhaps start with the foundational works on Indian linguistics, not only because linguistics is the earliest of Indian sciences to have been rigorously systematised, but also because this systematisation became the paradigm example for all other sciences.

It is said that

> काणादं पाणिनीयञ्च सर्वशास्त्रोपकारकम्।
>
> "The logic & epistemology and physics & metaphysics of Nyāya-Vaiśeṣika schools of philosophy, and the grammar of Pāṇini are helpful in understanding all Śāstras."

The *Aṣṭādhyāyī* of Pāṇini is also considered as a paradigmatic text, which serves as an ideal for all Śāstras

PĀṆINI AND EUCLID

"In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed ad hoc (i.e. Sūtras)....

Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in western thought. Scientific developments have therefore taken different directions in India and in the West.... In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. Just as Plato reserved admission to his Academy for geometricians, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics...."

[J.F.Staal, Euclid and Pāṇini, Philosophy East and West, 15, 1965, 99-116]

Note: The word "derived" means "demonstrated" in the case of Euclidean Geometry; it means "generated" in the case of Pāṇini's Grammar (Upapatti and Niṣpatti)

PATAÑJALI ON THE METHOD OF AṢṬĀDHYĀYĪ

Right at the beginning of his *Mahābhāṣya*, Patañjali raises the issue of how one can give an exposition of all the valid utterances

अथैतस्मिञ्शब्दोपदेशे सति किं शब्दानां प्रतिपत्तौ प्रतिपदपाठः कर्तव्यः गौरश्वः पुरुषो हस्ती शकुनिर्मृगो ब्राह्मण इत्येवमादयः शब्दाः पठितव्याः। नेत्याह। अनभ्युपाय एव शब्दानां प्रतिपत्तौ प्रतिपदपाठः। एवं हि श्रूयते बृहस्पतिरिन्द्राय दिव्यं वर्षसहस्रं प्रतिपदोक्तानां शब्दानां शब्दपारायणं प्रोवाच नान्तं जगाम।

"If this instruction of valid utterances is to be made for understanding the valid utterances, is a word-by-word list is to be made, (that is) words such as ‘cow’, ‘horse’, ‘man’, ‘elephant’, ‘bird’, ‘deer’, ‘brahmin’, should be listed? No, he says. For the instruction of valid utterances, this word-by-word listing of correct words is no good. Thus it is heard that, Bṛhaspati recited the valid utterances, by speaking word after word to Indra for thousand divine years, but could reach nowhere near the end."

[Mahābhāṣya of Patañjali, Paspaśāhnika]

PATAÑJALI ON THE METHOD OF AṢṬĀDHYĀYĪ

कथं तर्हीमे शब्दाः प्रतिपत्तव्याः। किञ्चित्सामान्यविशेषवल्लक्षणं प्रवर्त्यम्। येनाल्पेन यत्नेन महतो महतः शब्दौघान् प्रतिपद्येरन् ।

किं पुनस्तत् । उत्सर्गापवादौ । कश्चिदुत्सर्गः कर्तव्यःकश्चिदपवादः। कथंजातीयकः पुनरुत्सर्गः कर्तव्यः कथंजातीयकोऽपवादः। सामान्येनोत्सर्गः कर्तव्यः। तद्यथा। कर्मण्यण् (३। २ ।१)। तस्य विशेषेणापवादः। तद्यथा। आतोऽनुपसर्गे कः (३। २ ।३)।

"How are these utterances to be known?

Some characterisation (body of rules) containing the general and particular (conditions) is to be provided, by means of which [the students] are able to know, with a small effort, big, big string of utterances.

What is that characterisation? **Utsarga** (general rule) and **Apavāda** (special/exceptional rule)..."

[Mahābhāṣya of Patañjali, Paspaśāhnika]

कथं पुनर्ज्ञायते सिद्ध शब्दोऽर्थः सम्बन्धश्चेति।

लोकतः।

यल्लोकेऽर्थमर्थमुपादाय शब्दान्प्रयुञ्जते नैषां निर्वृत्तौ यत्नं कुर्वन्ति। ये पुनः कार्या भावा निर्वृत्तौ तावत्तेषां यत्नः क्रियते। तद्यथा घटेन कार्यं करिष्यन्कुम्भकारकुलं गत्वाह कुरु घटं कार्यमनेन करिष्यामीति। न तावच्छब्दान्प्रयुयुक्षमाणो वैयाकरणकुलं गत्वाह कुरु शब्दान्प्रयोक्ष्य इति। तावत्येवार्थमुपादाय शब्दान्प्रयुञ्जते। यदि तर्हि लोक एषु प्रमाणम् किं शास्त्रेण क्रियते।

लोकतोऽर्थप्रयुक्ते शब्दप्रयोगे शास्त्रेण धर्मनियमः ।

"The utterances, the meanings and the relation between them are established in the world....

One who needs to use pots goes to a potter and asks him to make pots for him. However, one who needs to use words, does not in the same way go to a grammarian and ask him to make words for him....

Since the meanings are established in the world, in the employment of words, the Śāstra lays down the proper discipline. "

[Mahābhāṣya of Patañjali, Paspaśāhnika]

PATAÑJALI ON THE METHOD OF AṢṬĀDHYĀYĪ

"In thus characterising grammar, Patañjali expounds perhaps the most essential feature of the Indian scientific effort. Science in India starts with the assumption that truth resides in the real world with all its diversity and complexity. For the linguist, what is ultimately true is the language as spoken by the people in all their diverse expressions. As Patañjali emphasises, valid utterances are not manufactured by the linguist, but are already established by the practice in the world. One does not go to a linguist asking for valid utterances, the way one goes to a potter asking for pots.

Linguists make generalisations about the language as spoken in the world. These generalisations are not the truth behind or above the reality of the spoken language. These are not idealisations according to which reality is to be tailored. On the other hand what is true is what is actually spoken in the real world, and some part of the truth always escapes our idealisation of it. There are always exceptions. It is the business of the scientist to formulate these generalisations, but also at the same time to be always attuned to the reality, to always be conscious of the exceptional nature of each specific instance. This attitude,... permeates all Indian science and makes it an exercise quite different from the scientific enterprise of the West."

J. K. Bajaj, Indian Tradition of Science and Technology 1988

BHRTṚHARI ON ŚĀSTRA AS UPĀYA (MEANS) WHICH ARE UNRESTRICTED (c. 500)

उपादायापि ये हेया तानुपायान् प्रचक्षते।
उपायानाञ्च नियमो नावश्यमवतिष्ठते ॥
अर्थं कथञ्चिद् पुरुष: कथञ्चित्प्रतिपद्यते। (वाक्यपदीयम् २.३८-९)

"Upāyas (procedures taught in Śāstras) are to be discarded, even though they are to be used for accomplishing an objective. There is no necessary limitation on such means. One accomplishes objectives by one means or the other."

As noted by the commentator Puṇyarāja:

कश्चिदाचार्यः पाणिनिविरचितेन लक्षणशास्त्रेण शब्दानधिगच्छति कश्चिदन्येनेति न नियम:।

"One preceptor (Ācārya) understands utterances by means of the grammatical framework of Pāṇini and another by means of another framework and thus there is no rule [that only a particular grammar is to be followed]."

Indian grammarians are not willing to commit to the uniqueness and universality of even Pāṇini's *Aṣṭādhyāyī*, whose efficacy is unsurpassed.

NAGEŚABHAṬṬA ON ŚĀSTRA AS UPĀYA (c. 1700)

तत्र वाक्यस्फोटो मुख्य: तस्यैव लोकेऽर्थबोधकत्वात्तेनैवार्थसमाप्तेश्चेति। ...तत्र प्रतिवाक्यं सङ्केतग्रहासम्भवाद् वाक्यान्वाख्यानस्य लघूपायेनाशक्यत्वाच्च कल्पनया पदानि प्रविभज्य पदे प्रकृतिप्रत्ययविभागान्प्रविभज्य कल्पिताभ्यामन्वयव्यतिरेकाभ्यां तत्तदर्थविभागं शास्त्रमात्रविषयं परिकल्पयन्ति स्माचार्या:।

तत्र शास्त्रप्रक्रियानिर्वाहको वर्णस्फोटः। प्रकृतिप्रत्ययास्तत्तदर्थवाचका एवेति तदर्थः। उपसर्गनिपातधात्वादिविभागोऽपि काल्पनिकः। स्थानिनो लादय आदेशास्तिबादयः कल्पिता एव। तत्र ऋषिभिः स्थानिनां कल्पिता अर्थाः कण्ठरवेणैवोक्ताः। आदेशानां तु स्थान्यर्थाभिदानसमर्थस्यैवादेशतेतिभाष्यात्तेऽर्था:। एवं च स्थानिनां वाचकत्वमादेशनां वेति विचारो निष्फल एव। कल्पितवाचकत्वस्योभयत्र सत्त्वात्।

मुख्यं वाचकत्वं तु कल्पनया बोधितसमुदायरूपे पदे वाक्ये वा लोकानां तत एवार्थबोधात्। '**उपेयप्रतिपत्त्यर्था उपाया अव्यवस्थिता**' इति न्यायेन व्याकरणभेदेन स्थानिभेदेऽपि न क्षतिः देशभेदेन लिपिभेदवदिति दिक्।

[नागेशकृत परमलघुमञ्जूषा, शक्तिनिरूपणम्]

NAGEŚABHAṬṬA ON ŚĀSTRA AS UPĀYA (c. 1700)

"There (among the Varṇa, Pada and Vākya-sphoṭas), it is the sentential meaning that is the primary; for it is the sentence which is seen to have import and completeness of meaning in the world....Since it is not feasible to identify all the (valid) sentences, and (mere consideration of sentences) will not provide any simple means for explaining sentence-meaning, the Ācāryas have devised a fictitious procedure, wherein sentences are divided into words and words into Prakṛti and Pratyaya and, following the procedure of Anvaya and Vyatireka (mutual presence and absence), they conceive of imputed meanings for these units only for the purpose of Śāstra (grammatical derivation).

There, the Varṇa-sphoṭa is for carrying out the procedures of Śāstra... Even the division into Upasarga, Nipāta, Dhātu etc., is fictitious. So are the substituends, Laṭ etc., and the substitutes, Tip etc....

Vācakatva (meaningfulness) rests mainly in the words or sentences which are made up of these imagined entities; for, in the world, only these (words and sentences) convey meanings. Indeed, following the well known principle that 'the Upāyas (grammatical derivations) are only for the realisation of the desired result and are otherwise unrestricted', there should be no cause of concern even if different substituends are employed in different grammars. It should be noted that this is akin to the fact that the scripts are found to vary from place to place."

THE EPISTEMOLOGY OF NYĀYA

Like the Science of Language (Śabdaśāstra), another discipline that is said to be at the basis of all the Śāstras is the logic and epistemology developed by the Nyāya school philosophy.

The Nyāya School recognises four Pramāṇas − means of acquiring valid knowledge: Pratyakṣa (perception), Anumāna (inference), Upamāna (analogy) and Śabda /Āgama (testimony/tradition).

The basic inferential scheme of traditional Nyāya is the following:

पर्वतो वह्निमान्।	The mountain has fire
धूमात्।	Because of smoke
य़त्र धूमस् तत्र वह्निर् यथा महानसे।	Where there is smoke there is fire, as in the kitchen
तथा चायम्।	This (mountain) is similar (has smoke)
तस्मात् तथा।	Hence this is similar (has fire)

This scheme is more like the scheme of inductive reasoning employed in sciences rather than the deductive syllogisms of Aristotle.

ANUMĀNA CANNOT CONTRADICT PRATYAKṢA AND ĀGAMA

Nyāya clearly specifies that inference should not be contrary to perception and tradition. In his commentary on *Nyāyasūtra*, Vātsyāyana says:

> कः पुनरयं न्यायः। प्रमाणैरर्थपरीक्षणं न्यायः। प्रत्यक्षागमाश्रितमनुमानं साऽन्वीक्षा। प्रत्यक्षागमाभ्यामीक्षितस्यान्वीक्षणमन्वीक्षा तया प्रवर्तत इत्यन्वीक्षकी न्यायविद्या न्यायशास्त्रम्। यत्पुनरनुमानं प्रत्यक्षागमविरुद्धं न्यायाभासः स इति।
>
> "What is this Nyāya? The examination of objects and ends by the means of true knowledge (Pramāṇas) is Nyāya. It is investigation (Anvīkṣā), which is inference that is subservient to perception and tradition. Anvīkṣakī, Nyāyā-vidyā or Nyāyaśātra is based on Anvīkṣā, investigation of that which has been seen through perception and tradition. Inference (Anumāna) which is contrary to perception or tradition is fallacious inference (Nyāyābhāsa)."
>
> *Nyāyasūtra-Vātsyāyanabhāṣya* 1.1

As we shall see later, the texts of Āyurveda also emphasise this important principle. They also emphasise that the text/tradition that are followed should not be contrary to perception.

ANUMĀNA IS NOT REASON

"In Western philosophical tradition, it was usual up until recent times, to ask: does knowledge arise from reason or from experience? The rationalists and empiricists gave different answers. These answers, in their various formulations, determined the course of Western philosophy. In Sanskrit philosophical vocabulary, the words ‘reason’ and ‘experience’ have no exact synonyms, and the epistemological issues were never formulated in such general terms....

That inference is different from reason (of the rationalists) is clear from the very etymology of the word *anumāna*; it follows upon perception. If we leave the Buddhists out, no school of Indian philosophy ascribed to reason a constructive role. It knows what can be known otherwise. There is always priority of perception. There are no Indian rationalists. Neither perception nor inference pointed to any specific faculty of mind − as ‘experience’ and ‘reason’ did in classical Western philosophies. The same faculties or cognitive instruments − operating in different manners − resulted in one case in perception, in another in inference."

J.N.Mohanty, *Reason and Tradition in Indian Thought*, Oxford 1992, pp.227-8

ANUMĀNA IS NOT REASON

"The Western concept of proof owes its origin to Plato's distinction between knowledge and opinion or between reason and sense. According to Plato, reason not merely knows objects having ontological reality, but also yields a knowledge which is logically superior to opinion to which the senses can aspire. On this distinction is based the distinction between contingent and necessary truths, between material truth and formal truth, between rational knowledge which can be proved and empirical knowledge which can only be verified....

As a matter of fact, the very concept of reason is unknown in Indian philosophy. In the systems which accept inference as a source of true knowledge, the difference between perception and inference is not explained by referring the two to two different faculties of the subject, sense and reason, but by showing that inferential knowledge is caused in a special way by another type of knowledge (Vyāpti-jñāna) whereas perception is not so caused (*jñānākaraṇakaṃ jñānam pratyakṣam*)..."

[Sibajiban Bhattacharya, The concept of proof in Indian mathematics and logic, in *Doubt, Belief and Knowledge*, Delhi, 1987, pp.193, 194, 197]

NYĀYA EXCLUDES FICTITIOUS ENTITIES FROM LOGICAL DISOCURSE

"Nyāya...[excludes] from logical discourses any sentence which will ascribe some property (positive or negative) to a fictitious entity. Vācaspati remarks that we can neither affirm nor deny anything of a fictitious entity, the rabbit's horn [Śaśaśṛṅga].

Thus Nyāya apparently agrees to settle for a superficial self-contradiction because, in formulating the principle that nothing can be affirmed or denied of a fictitious entity like rabbit's horn, Nyāya, in fact violates the same principle. Nyāya feels that this superficial self-contradiction is less objectionable [than admitting fictitious entities in logical discourse]...

By way of documentation... [we can refer to the *Asiddhi* section of] Udayana's *Ātmatattvaviveka*..."

[B.K.Matilal, *Logic Language and Reality*, Delhi 1985, p.94, 103-4]

"PROOFS" IN INDIAN MATHEMATICS

While there have been several extensive investigations on the history and achievements of the Indian mathematics, there has not been much discussion on the Indian mathematicians and philosophers' understanding of the nature and validation of mathematical results and procedures, their views on the nature of mathematical objects, and so on.

Traditionally, such issues have been dealt with in the detailed Bhāṣyas or commentaries, which continued to be written till recent times and played a vital role in the traditional scheme of learning. It is in such commentaries that we find detailed Upapattis or "proofs" of the results and procedures, apart from a discussion of methodological and philosophical issues.

In *Siddhāntaśiromaṇi*, Bhāskarācārya II (1150) presents the *raison d'être* of Upapatti in the Indian mathematical tradition:

मध्याद्यं द्युसदां यदत्र गणितं तस्योपपत्तिं विना
प्रौढिं प्रौढसभासु नैति गणको निःसंशयो न स्वयम् ।
गोले सा विमला करामलकवत् प्रत्यक्षतो दृश्यते
तस्मादस्म्युपपत्तिबोधविधये गोलप्रबन्धोद्यतः ॥

BHĀSKARA ON UPAPATTI (c.1150)

> "Without the knowledge of Upapattis, by merely mastering the calculations (Gaṇita) described here, from the *madhyamādhikāra* (the first chapter of *Siddhāntaśiromaṇi*) onwards, of the [motion of the] heavenly bodies, a mathematician will not be respected in the scholarly assemblies; without the upapattis he himself will not be free of doubt (Niḥsaṁśaya). Since Upapatti is clearly perceivable in the (armillary) sphere like a berry in the hand, I therefore begin the *Golādhyāya* (section on spherics) to explain the upapattis."

Thus, according to the Indian mathematical texts, the purpose of Upapatti is mainly:

(i) To remove confusion and doubts regarding the validity and interpretation of mathematical results and procedures; and,

(ii) To obtain assent in the community of mathematicians.

This is very different from the ideal of "proof" in the Greco-European tradition which is to irrefutably establish the absolute truth of a mathematical proposition.

UPAPATTI AND "PROOF"

The following are some of the important features of Upapattis in Indian mathematics:

1. The Indian mathematicians are clear that results in mathematics, even those enunciated in authoritative texts, cannot be accepted as valid unless they are supported by Upapatti. It is not enough that one has merely observed the validity of a result in a large number of instances.
2. Several commentaries written on major texts of Indian mathematics and astronomy present Upapattis for the results and procedures enunciated in the text.
3. The Upapattis are presented in a sequence proceeding systematically from known or established results to finally arrive at the result to be established.
4. In the Indian mathematical tradition the Upapattis mainly serve to remove doubts and obtain consent for the result among the community of mathematicians.
5. The Upapattis may involve observation or experimentation. They also depend on the prevailing understanding of the nature of the mathematical objects involved.

UPAPATTI AND "PROOF"

6. The method of Tarka or "proof by contradiction" is used occasionally. But there are no Upapattis which purport to establish existence of any mathematical object merely on the basis of Tarka alone. Thus the Indian mathematicians adopted a ‘constructivist’ approach to the existence of mathematical entities.
7. The Indian mathematical tradition did not subscribe to the ideal that Upapattis should seek to provide irrefutable demonstrations establishing the absolute truth of mathematical results.
8. There was no attempt made in Indian mathematical tradition to present the Upapattis in an axiomatic framework based on a set of self-evident (or arbitrarily postulated) axioms which are fixed at the outset.
9. While Indian mathematics made great strides in the invention and manipulation of symbols in representing mathematical results and in facilitating mathematical processes, there was no attempt at formalisation of mathematics.

STATUS OF PLANETARY MODELS IN INDIAN ASTRONOMY

In his *Āryabhaṭīyabhāṣya* (c.629), while explaining the planetary model of Āryabhaṭa, Bhāskara I notes that notions such as the apsides (Ucca, Nīca), mean (Madhyama), epicycles (Paridhi) etc., are conceptual tools which serve the purpose of arriving at the observed motion of planets and there are no constraints on them except that they should lead to observed results.

उच्चनीचमध्यमपरिधिरित्येवमादिस्फुटगतिसाधनोपाभूतानाञ्च उपायानां नैव नियमोक्तिर्वा विद्यते। केवलं तु उपेयसाधका उपायाः। तस्मादियं सर्वा प्रक्रिया असत्या यया ग्रहाणां स्फुटगतिः साध्यते।... तथा हि ...शाब्दिकाः प्रकृतिप्रत्ययविकारागम-वर्णलोपव्यत्ययादिभिः शब्दान् प्रतिजानीते। एवमत्रापि मध्यममन्दोच्चशीघ्रोच्च-तत्परिधिज्याकाष्ठभुजाकोटिकर्णादिव्यवहारेण सांवत्सरा ग्रहाणां स्फुटगतिं प्रतिजानते। तस्मादुपायेष्वसत्येषु सत्यप्रतिपादनपरेषु न चोद्यमस्ति।

> "There are no constraints or limitations imposed on the notions such as Ucca, Nīca, Madhyama, Paridhi and so on, which are essentially aids to the calculation of the observed motion of the planets. They are only the means for arriving at the desired results. Hence this entire procedure is fictitious, by means of which the observed planetary motion is arrived at....Just as the linguists utilise notions such as Prakṛti, Pratyaya, Vikāra, Āgama, Varṇa, Lopa, Vyatyaya, etc., to comprehend (well-formed) words.... In the same way in our science also astronomers employ notions such as Madhyama, Mandocca, Śīghrocca, Śīghra-paridhi, Jyā, Kāṣṭha, Bhujā, Koṭi, Karṇa, etc., in order to comprehend the observed motion of planets. Hence, there is indeed nothing unusual that fictitious means are employed to arrive at the true state of affairs [in all these Śāstras]."

Thus, like the Grammarians, the Indian Astronomers were in the business to calculate and to compute, not to form pictures of the heavens as they ought to be. Indian astronomers do employ various models, analytical as well as geometrical, but the texts themselves emphasise, these are no more than artefacts in their calculations.

CONTINUOUS EXAMINATION IS MANDATED BY THE ŚĀSTRA

Devācārya (c.689) on why he is attempting a new Karaṇa (manual for computing planetary motions):

शिष्यस्य बुद्धिमान्द्यादाचार्यस्योपदेशसंवरणात्।
गुणभागयोश्च शेषात् पुराणकरणानि न घटन्ते॥
नष्टानि स्थापयितुं नवानि करणानि च प्रकाशयितुं ।
तन्त्रज्ञानस्य फलं वदन्ति तदयं ममोत्साहः॥

"The Karaṇas (astronomical manuals) of ancient times do not yield accurate results either because of the dullness of the pupil's intellect, or because of the cryptic instruction of the teacher, or because of the inexactitude of the multipliers and divisors. It is said that the aim of acquiring knowledge of the Śāstra is to rectify and reestablish Karaṇas so lost or to bring to light new ones. Hence this attempt of mine."

Devācārya's *Karaṇaratna* 1.3-4

CONTINUOUS EXAMINATION IS MANDATED BY THE ŚĀSTRA

While concluding his *Āryabhaṭīyabhāṣya*, Nīlakaṇṭha refers to the fact that Parameśvara (c.1430) came up with his revisions after making careful observations for 55 years, and states:

परमेश्वराचार्येण पुनर्ग्रहणग्रहयोगादिकं यन्त्रैः पञ्चपञ्चाशतद्वर्षकालं सम्यग् परीक्षितम्।...तस्य गोलवित्तमत्वं च गोलदीपिकादिभिः तत्कृतैः प्रबन्ध-विशेषैर्ज्ञायते। अतोऽन्येषां करणानां स्वस्वकाले यावत्सूक्ष्मत्वं ततोऽप्येतत्कृतस्य दृग्गणिताख्यस्य सूक्ष्मतमत्वमुपपद्यते। ... अतः परीक्ष्यैव सर्वैरपि शिष्येभ्यः उपदेष्टव्यम्। तदा सति न विसंवादः कदाचिदपि भविष्यति। जायमानस्यापि परिहार्यत्वात् ।

"For a period of fifty-five years, Parameśvarācarya carefully observed eclipses, conjunction of planets, etc., by means of instruments. ... His mastery of spherical astronomy is also known by the special monographs that he wrote such as *Goladīpikā*, etc. Just as the Karaṇas composed by others were accurate during their times, his composition *Dṛgganita* is indeed most accurate... Therefore, everyone should instruct their students only after due examination. If so, there will not be any discrepancy [between the instruction and observations]. Even those that may arise, are in fact correctible [by this method]. "

PURPOSE OF ŚĀSTRA: TO IMPART THE METHOD OF EXAMINATION

Nīlakaṇṭha also wrote a tract, *Jyotirmīmāṁsā*, highlighting the role of examination (Parīkṣā) in Astronomy. He concludes that the very purpose of the Śāstra is to enable the students to acquire the capacity to examine the motion of celestial bodies.

नन्वेवमपि स्वकाल एव गीतिकोक्तभगणाद्याः [सूक्ष्मा: यदा] गीतस्य ग्रहणस्य च प्रत्यक्षसंवादः स्यात् यत इदानीं ग्रन्थकरणकालात् तृतीये दिव्याब्दे महान् भेद उपलभ्यते। गीतिकोक्तकालतः पश्चादेव हीदानीं सर्वाण्यपि ग्रहणानि दृश्यन्ते।...

एवमादिदूषणं परैरुद्भाव्यमानं परिहर्तुं परीक्षाप्रकारमाह यदर्थं पदत्रयेण सकला युक्तयः प्रदर्शिताः 'क्षितिरवियोगाद् दिनकृद् रवीन्दुयोगाद्' इति अत्रोक्ताभिर्युक्तिभिरेव बुद्धिमद्भिः सम्यक् परीक्षणं शक्यं कर्तुम्।

ननु तपोभिः प्रसन्नो ब्रह्मा आर्यभटाय भगणपरिध्यादिकं ग्रहगणनसाधनभूतं संख्याविशेषमुपदिदेश। तदुपदिष्टं पुनरार्यभटः सर्वं यथोपदिष्टमेव दशभिर्गीतिभिः निबबन्ध इति केचिन्मन्यन्ते। तस्य कुतः परीक्षणं ब्रह्मणः सर्वज्ञत्वात् रागद्वेषाद्यभावाच्च अविततत्त्वनिश्चयात् इति चेत् मन्द मैवम्। देवताप्रसादो मतिवैमल्यहेतुरेव। न च पुनः ब्रह्मा आदित्यो वा स्वयमेवागत्य उपदिशेत्। एवमेव च वक्ष्यति चानन्तरसूत्रे 'सदसज्ज्ञानसमुद्रात् समुद्धृतं देवताप्रसादेन। सज्ज्ञानोत्तमरत्नं मया निमग्नं स्वमतिना वा॥' इति...

PURPOSE OF ŚĀSTRA: TO IMPART THE METHOD OF EXAMINATION

तस्मात् शिष्यप्रशिष्यपरंपरया सर्वैरपि परीक्षणं कार्यम् ।...

आर्यभटीयस्य च परीक्षापरत्वादेव सकलदेशकालयोः स्फुटार्थत्वं न पुनः तदुक्तभगणादिवैशिष्ट्यात्। अत इदमेव परीक्षासूत्रं सिद्धान्तान्तरेभ्योऽस्य गौरवमापादयति ।

मानसव्याख्यातापि कश्चिदाह 'ननु पैतामहादिभेदेन परस्परविरुद्धाश्च सिद्धान्ता भवन्ति। सिद्धान्तभेदे सति कालभेद:। कालभेदे सति कालाङ्गानि श्रौतस्मार्तलौकिकानि कर्माणि विफलानि स्युः। कर्मवैकल्ये सति लोकयात्रोच्छेदः। हा धिक् सङ्कटे महति पतिता: स्म:।'

अत्रोच्यते 'ऋजुमते स न शोचितव्य:। गुरुचरणपरिचरणपरैः किमिव न ज्ञायते। पञ्चसिद्धान्तास्तावत् क्वचित्काले प्रमाणमेव इत्यवगन्तव्यम्। अपि च यः सिद्धान्तः दर्शनविसंवादी भवति सोऽन्वेषणीयः। दर्शनसंवादश्च तदानीन्तनैः परीक्षकैर्ग्रहणादौ विज्ञातव्यः। ये पुनरन्यथा प्राक्तनसिद्धान्तस्य भेदे सति यन्त्रैः परीक्ष्य ग्रहाणां भगणादि ज्ञात्वा अभिनवसिद्धान्त: प्रणेय इत्यर्थात् तत् त इहलोकेऽहसनीया: परलोकेऽदण्डनीयाश्च'इति ।...

तस्मात् शिष्याणां ग्रहगतिसामर्थ्यापादनमेव शास्त्रप्रयोजनम्। ते पुनः दृक्संवादिकरणं कृत्वा लोके सञ्चरेयुः। करणनामेव हि व्यावहारिकत्वं सूक्ष्मत्वं च स्यात्।

PURPOSE OF ŚĀSTRA: TO IMPART THE METHOD OF EXAMINATION

"The number of revolutions etc., enunciated in the *Gītikā* [*pāda* of *Āryabhaṭīya*], are accurate only at the time of its composition, when they would have been tested for consonance with eclipses etc. Currently, in the third divine year [of 360 years each] after the composition of the text, one finds great differences [between calculations and observations]. All eclipses are now seen at times later than those computed [according to *Āryabhaṭīya*].

It is only to lay at rest such criticism, which is bound to be made by others, that [Āryabhaṭa] gave the method of examination (Parīkṣāprakāra), all the techniques of which are expressed merely by the three words 'The Sun by the conjunction of the earth and Sun; by the conjunction of Sun and the Moon [the Moon is ascertained].' By following these techniques only, the wise can do proper examination.

PURPOSE OF ŚĀSTRA: TO IMPART THE METHOD OF EXAMINATION

"Some people indeed believe that, pleased by his penance (Tapas), Brahmā instructed Āryabhaṭa the number of revolutions, [dimensions of] epicycles etc., which are to be employed in calculating the motion of planets; and that Āryabhaṭa encapsulated all that instruction faithfully in ten Gītikā verses. And so, [you may argue], how can we conceive of putting that [instruction] to test, since Brahmā is indeed omniscient and free from all mental aberrations such as attachment, hatred etc., and is certainly free of error?

You dim-witted, it is not so. The grace of gods is only for attaining clarity of intellect. Again it cannot be that Brahmā or Sun would come himself and instruct. In fact [Āryabhaṭa] states more or less the same in a later verse:

‘By the grace of Brahmā, the precious jewel of excellent knowledge [of Jyotiṣa] has been brought out by me by means of the ship of my intellect from the sea of true and false knowledge by diving deep into it.’...

PURPOSE OF ŚĀSTRA: TO IMPART THE METHOD OF EXAMINATION

"Therefore Parīkṣā (examination) is to be done continuously, following the tradition of disciples and their disciples etc., by all...

It is only because *Āryabhaṭīya* has enunciated the supremacy of Parīkṣā, that it is a relevant and valid text for all places and times, and not because of any specialty of the revolution numbers and other parameters stated therein. It is this rule of Parīkṣā which gives it an exalted status in relation to other Siddhāntas....

A commentator on the *Mānasa* [*Laghumānasa* of Muñjāla] has lamented: 'Indeed, the Siddhāntas, like *Paitāmaha*, differ from one another [in giving the astronomical constants]. Timings are different as the Siddhāntas differ [i.e. the measures of time for any particular event as computed by the different Siddhāntas]. When the computed timings differ, Vedic and domestic rituals, which have [correct] timings as a component [of their performance] go astray. When rituals go astray, worldly life gets disrupted. Alas, we have precipitated into a calamity.'

PURPOSE OF ŚĀSTRA: TO IMPART THE METHOD OF EXAMINATION

"Here, it needs to be stated: ‘O faint-hearted, there is nothing to be despaired of. Wherefore does anything remain beyond the ken of those intent on serving at the feet of the teachers? One has to realise that the five Siddhāntas had been valid at a particular time. Therefore, one should look for a Siddhānta that does not show discord with actual observations [at the present time]. Such accordance with observation has to be ascertained by astronomers during times of eclipses etc. When earlier Siddhāntas show discord, observations should be made with instruments and revolutions etc. obtained, [which would give results which accord with actual observation] and a new Siddhānta enunciated. Thus, nobody will be ridiculed in this world nor punished in the next.’...

Therefore, the purpose of the Śāstra is to create in students the capacity for examining the motion of the Grahas. They, in turn, should function in the world by composing a Karaṇa [computational manual for their epoch] which is in accordance with observations. Only such Karaṇas can be accurate and of use in worldly affairs."

ĀYURVEDA'S EMPHASIS ON YUKTI

"Like in linguistics and astronomy, the remarkable feature of Indian tradition of medicine is its pragmatic attitude towards scientific theorisation. The Āyurvedic texts provide a theoretical framework through which the problem of finding an appropriate cure for a particular patient must be approached. However, the texts never tire of reminding the practitioner that he must never be guided by mere theoretical considerations, and therefore he must be constantly observant of all the specific features that a particular case presents.

For Caraka Saṃhitā the most desirable intellectual accomplishment of a doctor is that of possessing Yukti, which is defined as the capacity of the trained intellect to see the course of action through the complexity of phenomena with their multiple causes."

J. K. Bajaj, Indian Tradition of Science and Technology 1988

CARAKA ON THE IMPORTANCE OF YUKTI

द्विविधं खलु सर्वं सच्चासच्च तस्य चतुर्विधा परीक्षा आप्तोपदेशः प्रत्यक्षं अनुमानं युक्तिश्चेति।...

जलकर्षणबीजर्तुसंयोगात् सस्यसंभवः। युक्तिः षड्धातुसंयोगाद्गर्भाणां संभवस्तथा॥
मथ्यमन्थकमन्थानसंयोगादग्निसंभवः। युक्तियुक्ता चतुष्पादसम्पद्व्याधिनिबर्हिणी॥

(सूत्रस्थानम् ११.१७,२३, २४)

"Indeed all that (can be investigated) is of two kinds: Sat and Asat. There are four ways of investigating them: The teaching of Āptas, perception, inference and Yukti....

[The knowledge that] crops come about by the proper conjunction of water, ploughing [of the field], seed and seasons, the creation of the foetus is possible by the proper conjunction of six Dhatus, fire comes about by the proper conjunction of fire-stick, base and friction are [all instances of exercise of] Yukti. The four pillars of treatment [physician, materials, attendant, patient] of good quality, and [the physician endowed] with Yukti, will surely lead to the cure of diseases."

CARAKA ON THE IMPORTANCE OF YUKTI

Cakrapāṇi in his commentary explains that, since Yukti is not well-known in other Śāstras (such a Nyāya) as a means of knowledge, Caraka has first given a few examples so that it will be easier to follow the definition of Yukti as given in the next verse:

> बुद्धिः पश्यति या भावान् बहुकारणयोगजान्।
> युक्तिस्त्रिकाला सा ज्ञेया त्रिवर्गः साध्यते यया॥ (सूत्रस्थानम् ११.२५)
>
> "The intellect which perceives the things that come about by a proper conjunction of several causes, is to be understood as Yukti, which [like Anumāna or inference] operates over all the three Kālas (past, present and future), and through which all the three goals of life can be realised."

Cakrapāṇi explains that, though Yukti is not an independent separate means of knowledge, it is a faculty which assists the other means of knowledge, and plays a crucial role in understanding of effects in relation to the multiple causative factors responsible for them.

ANUMĀNA CANNOT CONTRADICT PRATYAKṢA AND ĀGAMA

The *Suśrutasaṁhitā*. while defining the theoretical categories through which the medicinal properties of a substance are to be determined warns that the wise physician should never raise theoretical arguments about the properties of a drug when they are already known and established in tradition based on actual practice.

प्रत्यक्षलक्षणफला: प्रसिद्धाश्च स्वभावतः।
नौषधीर्हेतुभिर्विद्वान् परीक्षेत कथञ्चन॥
सहस्रेणापि हेतूनां नाम्बष्ठादिर्विरेचयेत्।
तस्मात्तिष्ठेत्तु मतिमानागमे न तु हेतुषु॥

[सुश्रुतसंहिता ४०.२०,२१]

"An expert physician should never doubt through reasons (Hetus) the efficacy of drugs which have been established in practice and whose nature is well known. Even thousands of reasons will not force the Ambaṣṭha group of drugs into purgative action. So, the wise physician should stick to established tradition and not get lost in reasons."

ĀGAMA CANNOT CONTRADICT PRATYAKṢA

In the same way, the Āyurvedic texts emphasise that whatever has been carefully observed in one's practise should take precedence over what is mentioned in texts. The *Śarṅgadharasaṁhitā* declares that what is observed to be ineffective in practise should be avoided even if it is mentioned as a part of the relevant group of remedies.

> व्याधेरयुक्तं यद्द्रव्यं गणोक्तमपि तत्त्यजेत्
> अनुक्तमपि यद्युक्तं योजयेत्तत्र तद्बुधः
>
> शाङ्र्गधरसंहिता १.१.५३

> "A drug, although mentioned [in the texts] as a member of a group (Gaṇa) [regarded as useful in a particular disease] should never be prescribed for that disease if it is observed to be ineffective. Even if it is not mentioned [as a member of the relevant group] a drug which is observed to be effective should be prescribed for that disease by a wise physician."

THE WHOLE WORLD IS A TEACHER FOR THE WISE

With their pragmatic approach to theorisation, the Āyurvedic texts display a refreshing openness. They are acutely aware that the cowherds, the shepherds, the forest-dwellers, etc., do know most of the medicines by name and form (*Carakasaṁhitā, Sūtrasthāna* 1.120-3), and call upon the physician to build on this knowledge. The *Vimānasthāna* of *Carakasaṁhitā* contains a long discourse given by the teacher while initiating a new student, at the end of which the teacher says:

> न चैव ह्यस्ति सुतरामायुर्वेदस्य पारं तस्मादप्रमत्तः शश्वदभियोगमस्य गच्छेत् एतच्च कार्यं एवं भूयश्च वृत्तसौष्ठवमनसूयता परेभ्योऽप्यागमयितव्यं कृत्स्नो हि लोको बुद्धिमतामाचार्यः शत्रुश्चाबुद्धिमताम् ...
>
> "The science of Āyurveda is indeed limitless. Hence constantly engage in this science with due diligence. And this should also be done. Without any sense jealousy one should learn laudable practices from others also. The whole world is a teacher for the wise and an enemy for the foolish." [Caraka, Vimānasthāna, 8.14]

THE ŚĀSTRA ONLY ILLUMINATES

The Āyurvedic texts are very clear that the Śāstra so to say is only a light which illuminates, and it is the intellect of the physician which has to come into play to perceive the proper course of action. After listing the various qualities of a good physician, Caraka states:

शास्त्रं ज्योतिः प्रकाशार्थं दर्शनं बुद्धिरात्मनः।
ताभ्यां भिषक् सुयुक्ताभ्यां चिक्त्सन्नापराध्यति॥

"The Śāstra is a light which serves to illuminate. It is once own intellect that really perceives [the course of action]. A physician who is well endowed with both will never err in his treatment."

[Caraka, Sūtrasthāna, 9.24]

"... [Maintaining a] pragmatic attitude towards scientific theorization, made the doing of science in India a rather painstaking business. The Indian scientists, not having the luxury of reducing the reality of the world to that encompassed by their theories of the time, had to be continuously aware of the world in its complete complexity, and had to continuously refine and simplify their procedures in order to operate successfully within the complexity of the world.

That they were able to do this systematically in a number of fields over a long period of more than 2,500 years is a measure of their ingenuity and industry. One can only marvel at the enormity of the task of encapsulating the whole of Sanskrit language as it was spoken in 4,000 aphoristic rules.

Equally remarkable are the efforts of the astronomer-mathematicians of repeatedly refining their parameters to fit the observations, so that ever since Āryabhaṭa the Indians always had access to reasonably accurate information about the motions of the heavens. But the astronomer-mathematicians also simplified their computations to an extent that learned Brahmins in their innumerable locales could also compute all the astronomical information that mattered to the residents.

INDIAN TRADITION OF SCIENCE

"The effort of Indian physicians falls in the same class. They were not only able to painstakingly acquire and systematise within their theoretical framework all the information about drugs and diseases that was current amongst the people in diverse areas, but were also able to simplify their theories sufficiently so that much of the Āyurvedic science became the folklore of health known in all families. The fact that the Indian scientists given their theoretical attitude had to be necessarily open to the world around them perhaps ensured that the folk and the science continued to remain in a symbiotic relation with each other.

Besides linguistics, astronomy and mathematics, and medicine, Indians also developed the sciences of matter (Padārthaśāstra), chemistry (Rasaśāstra), architecture (Vāstuśāstra), music (Saṅgītaśāstra) etc. To all of these sciences they brought their peculiarly Indian mode of careful but tentative generalisation and continuous keen observation."

J. K. Bajaj, Indian Tradition of Science and Technology 1988

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