We recall that in a magic square, the numbers in the rows, columns and diagonals sum to the same magic total. In a pan-diagonal magic square, the broken diagonals also yield the same magic total. Succinctly put, pan-diagonal magic squares have the remarkable property that they can be considered as a magic squares "on the torus". It is of interest to note that Rosser and Walker proved in 1936 (the proof was simplified by Vijayaraghavan in 1941) that there are only 384 pan-diagonal 4×4 magic squares with entries $1, 2, \ldots, 16$.

Curiously enough, Ramanujan, in his notebooks of probably his earliest school days, has the magic square

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

This turns out to be one of the 384 magic squares considered by Narayana Pandita. (Notice that rows, columns, diagonals and broken diagonals add to 34. For example, $5 + 3 + 12 + 14 = 12 + 9 + 5 + 8 = 34$).

Xenophanes, the founder of the Eleatic School of Philosophy, had the well known dictum – Ex nihilo nihil fit, "Out of nothing, nothing comes". One wonders whether after all Ramanujan was indeed influenced somewhat by the mathematical tradition of his ancestors.

Kerala School of Astronomy and Mathematics

M. D. Srinivas

Centre for Policy Studies, Chennai

E-mail: mdsrinivas50@gmail.com

In the first quarter of the nineteenth century, Benjamin Heyne and Charles Whish, officers serving under the East India Company, came across traditional practitioners of Indian astronomy who seemed to be conversant with several infinite series for the ratio of the circumference to the diameter of a circle (denoted by the Greek symbol π in modern mathematics) and for the trigonometric functions sine and cosine. Such series were generally believed to have been discovered first in Europe by Gregory, Newton and Leibniz in the second half of the 17th century.

Heyne's observations were recorded in the work *Kalasankalita* published in 1825 CE by John Warren of the Madras Observatory. Charles Whish read a paper before the Madras Literary Society in 1832, which was subsequently published in the Transactions of the Royal Asiatic Society, where he referred to four important works of the Kerala School, *Tantrasangraha, Yuktibhasha, Karanapaddhati* and *Sadratnamala*, which contained these infinite series. Whish also promised to outline the very interesting proofs of these results contained in the Malayalam work *Yuktibhasha*, though he died soon thereafter before fulfilling the promise.

Whish's article was widely taken note of in the European scholarly circles of that period, but the message was soon forgotten. Most of the scholarly works on Indian astronomy and mathematics, at least till the last quarter of the twentieth century, continued to maintain that the Creative period in Indian mathematics ended with Bhaskaracharya II (c.1150 CE) and that Indian mathematics, bereft of any notion of proof, had no logical rigour. The monumental work of great scholars such as Ramavarma Maru Thampuran, C.T. Rajagopal, K.V. Sarma, and their collaborators in the last sixty years, has largely dispelled such misconceptions and brought about a new perspective on the historiography of Indian mathematics in the medieval era. Recently, critical editions of *Yuktibhasha, Tantrasangraha* and *Sadratnamala* have been published along with English translation and detailed mathematical notes[1].

According to some scholars, the great Aryabhata hailed from Kerala, though he wrote his treatise *Aryabhatiya* at Kusumapura or modern Patna in 499 CE. Kerala is of course known for many important astronomers such as Haridatta (c.650–700) the originator of the well-known *Parahita* system

[1] *Ganita-Yuktibhasha of Jyesthadeva*, Critically Edited and Translated with Explanatory Notes by K.V. Sarma, K. Ramasubramanian, M.D. Srinivas and M.S. Sriram, Hindustan Book Agency, New Delhi 2008 (Rep. Springer, New York 2009); *Tantrasangraha of Nilakantha Somayaji*, Translated with Explanatory Notes by K. Ramasubramanian and M.S. Sriram, Springer NewYork 2011; *Sadratnamala of Sankaravarman*, Translated with Explanatory Notes by S. Madhavan, KSR Institute, Chennai 2011.

(founded at Tirunavay in 683), Govindasvamin (c.800), Sankaranarayana (c.825–900) and Udayadivakara (c.1073), but it was Madhava (c.1340–1425) of Sangama Grama (near present-day Ernakulam) who was the pioneer of a new School. The later members of the school, starting from Madhava's disciple Paramesvara (c.1380–1460), lived mostly around the river Nila or Bharatapuzha in South Malabar.

After Madhava, the next important member of the school was Nilakantha Somasutvan (c.1444–1550) of Trikantiyur, who was the disciple of Damodara, the son of Paramesvara. Another disciple of Damodara was Jyeshthadeva (c.1500–1610), the author of the celebrated work *Yuktibhasha* in Malayalam. The line of direct disciples of Madhava continued up to Acyuta Pisarati (c.1550–1621), a student of Jyesthadeva and the teacher of the great scholar devotee Narayana Bhattatiri. By the time of Acyuta, Malabar had become a highly disturbed region – the scene of constant warfare between rival European powers, the Portuguese and the Dutch. However, the Kerala School managed to survive well into the nineteenth century, when Sankaravarman the Raja of Kadattanadu wrote perhaps the last important work emanating from the School, *Sadratnamala*, in 1819 CE.

Only a couple of astronomical works of Madhava – he is always referred to as *Golavid* (master of spherics) by his disciples – have come down to us. These works, *Venvaroha* and *Sphutachandrapti*, do reveal Madhava's great mathematical skill in improving the accuracy of the ingenious *vakya* system of computation for the Moon. Madhava's important results on infinite series are known through his verses cited in the later works of Nilakantha, Jyesthadeva, Sankara Variar and others. While enunciating the following infinite series (the so called Gregory-Leibniz series) for the ratio of the Circumference (C) to the diameter (D) of a circle,

$$C/4D = 1 - 1/3 + 1/5 - 1/7 + \cdots,$$

Madhava notes that accurate results can be obtained by using end-correction terms, for which he gives successive approximations. Thus, according to Madhava, if one gets tired after summing n terms in the above series, he may use the following approximation involving an end-correction term which is said to be "highly accurate":

$$\begin{aligned} C/4D \approx\ & 1 - 1/3 + 1/5 - 1/7 + \cdots \\ & + (-1)^{(n-1)}[1/(2n-1)] + (-1)^n[n/(4n^2+1)] \end{aligned}$$

If we sum fifty terms in the series ($n = 50$) and use the above end-correction term, we obtain a value of $\pi = C/D$, accurate to eleven decimal places. A verse of Madhava, giving the value of π to the same accuracy, is cited by Nilakantha in his commentary on *Aryabhatiya*. Based on the technique of end-correction terms, Madhava also presents several transformed series for π which involve higher powers of the denominator, such as the following:

$$\begin{aligned} C/16D = 1/(1^5 + 4 \times 1) - 1/(3^5 + 4 \times 3) \\ + 1/(5^5 + 4 \times 5) - \cdots \end{aligned}$$

Thus, Madhava is not only a pioneer in discovering an exact infinite series for π, he is also a trail blazer in discovering rapidly convergent approximations and transformations of the series. In the same way, Madhava is not satisfied with merely enunciating the infinite series for the sine and cosine functions (the so called Newtons series), he also comes up with an algorithm (in terms of the famous mnemonics *"vidvan"*, *"tunnabala"*, etc.) for evaluating these functions accurately (correct to five decimal places) for an arbitrary argument.

Detailed justifications (called *upapatti* or *yukti* in Indian mathematical tradition) for the results discovered by Madhava are presented in the famous Malayalam work *Yuktibhasha* (c.1530), which is perhaps the "First Textbook on Calculus". In demonstrating these infinites series, *Yuktibhasha* proceeds systematically by first deriving the binomial expansion

$$(1+x)^{-1} = 1 - x + x^2 + \cdots + (-1)^r x^r + \cdots$$

Yuktibhasha then presents a demonstration (by induction) of the following estimate for the sum of powers of natural numbers when n is large:

$$1^k + 2^k + \cdots + n^k \approx n^{k+1}/(k+1)$$

The above estimate was rediscovered by various mathematicians in Europe in the 17th century and it played a crucial role in the development of calculus. While deriving the successive approximations to the end-correction term, *Yuktibhasha* employs a method analogous to approximating a continued fraction by its successive convergents. Incidentally, we may remark that, a few centuries later, Srinivasa Ramanujan was to display great facility in transforming series and continued fractions to obtain many spectacular results.

Any account of the work of the Kerala School cannot fail to mention its important achievements in Astronomy. After all Madhava's accurate sine tables, interpolation formulae etc. were meant for applications in Astronomy. Madhava's disciple Paramesvara is reputed to have made careful observations over a period of fifty-five years and he came up with the *Drigganita* system. However, it was Nilakantha who arrived at a major revision of the traditional planetary theory, which can be characterised as a landmark in the history of Astronomy.

In the Indian astronomical tradition, at least from the time of Aryabhata (c.499 CE), the procedure for calculating the geocentric longitudes of the planets, Mercury, Venus, Mars, Jupiter and Saturn involved essentially the following steps. First, the mean longitude was calculated for the desired day and then two corrections, namely the *manda-samskara* and *sighra-samskara*, were applied to the mean planet to obtain the true longitude. In the case of the exterior planets, Mars, Jupiter and Saturn, the *manda*-correction is equivalent to taking into account the eccentricity of the planet's orbit around the Sun and the *manda*-correction coincides, to first order in eccentricity, with the equation of centre currently calculated in astronomy. This is followed by the *sighra-samskara*, which (in current parlance) is equivalent to converting the heliocentric longitude into the geocentric longitude.

In the case of interior planets, Mercury and Venus, the traditional planetary model was not successful in capturing their heliocentric motion. The *manda*-correction was applied to the mean Sun, instead of the mean heliocentric planet. But this was an error that was common to all the ancient planetary models developed in the Greek, Islamic and European traditions till the time of Kepler. The Indian planetary models, however, gave a fairly correct procedure for the calculation of latitudes of the interior planets based on their notion of *sighrocca*. This fact that there were two different procedures for the computation of planetary latitudes was noted as an unsatisfactory feature of the traditional planetary theory by Bhaskaracarya II (c.1150) and many others.

Nilakantha resolved this long-standing problem by proposing a fundamental revision of the traditional planetary theory. In his treatise *Tantrasangraha* (c.1500), Nilakantha proposed that in the case of Mercury and Venus, the *manda*-correction or the equation of centre should be applied to what was traditionally identified as the *sighrocca* of the planet – which, in the case of interior planets, corresponds to what we currently refer to as the mean-heliocentric planet. This led to a more accurate formulation of the equation of centre and the latitudinal motion of the interior planets than was available either in the earlier Indian works or in the Greco-European or the Islamic traditions of astronomy till the work of Kepler, which was to come more than a hundred years later. (Incidentally, it may be noted that the celebrated works of Copernicus (c.1543) or Tycho Brahe (c.1583) did not bring about any improvement in the planetary theory of interior planets as they merely reformulated the ancient planetary model of Ptolemy for different frames of reference.) In fact, in so far as the computation of the planetary longitudes and latitudes is concerned, Nilakantha's revised planetary model closely approximates the Keplerian model, except that Nilakantha conceives of the planets as going in eccentric orbits around the mean Sun rather than the true Sun.

Nilakantha was also the first savant in the history of astronomy to clearly deduce from his computational scheme and the observed planetary motion (and not from any speculative or cosmological argument) that the interior planets (Mercury and Venus) go around the Sun and the period of their motion around Sun is also the period of their latitudinal motion. He explains in his commentary on the *Aryabhatiya* that the Earth is not circumscribed by the orbit of Mercury and Venus; and the mean period of motion in longitude of these planets around the Earth is the same as that of the Sun, precisely because they are being carried around the Earth by the Sun.

In his works, *Golasara, Siddhantadarpana*, and a short but remarkable tract *Grahasphutanayane Vikshepavasana*, Nilakantha describes the geometrical picture of planetary motion that follows from his revised planetary theory, according to which the five planets Mercury, Venus, Mars, Jupiter and Saturn move in eccentric orbits (inclined to the ecliptic) around the mean Sun, which in turn goes around the Earth. (This geometrical picture is the same as the model of solar system proposed in 1583 CE by Tycho Brahe, albeit on entirely different considerations.) Most of the Kerala astronomers who succeeded Nilakantha, such as Jyeshthadeva, Acyuta Pisarati, Putumana Somayaji, etc. seem to have adopted his revised planetary model.

Ever since the seminal work of Needham, who showed that till around the sixteenth century Chinese science and technology seem to have been more advanced than their counterparts in Europe, it has become fashionable for historians of science to wonder "Why modern science did not emerge in non-western societies?" In the work of the Kerala School, we notice clear anticipations of some of the fundamental discoveries which are associated with the emergence of modern science, such as the mathematics of infinite series and the development of new geometrical models of planetary motion. It seems therefore more appropriate to investigate "Why modern science ceased to flourish in India after the 16th Century?" It may also be worthwhile to speculate "What would have been the nature of modern science (and the modern world) had sciences continued to flourish in non-western societies?" In this way we could gain some valuable insights regarding the sources and the nature of creativity of geniuses such as Srinivasa Ramanujan, Jagadish Chandra Bose and others in modern India.

Interview with Srinivasa Varadhan

R. Sujatha

Srinivasa Varadhan*

Srinivasa Varadhan, known also as Raghu to friends, was born in Chennai (previously Madras) in 1940. He completed his PhD in 1963 in the Indian Statistical Institute (ISI), Calcutta, and has been in Courant Institute of Mathematical Sciences since 1963. An internationally renowned probabilist, he was awarded the Abel Prize in 2007 and was honoured with the National Medal of Science by President Obama in 2010. Sujatha Ramdorai followed up an e-interview with a free wheeling conversation in Chennai on January 10, 2012, where he spoke on subjects ranging from his career and mathematics to science education and mathematical talent in Asia.

Sujatha Ramdorai: Congratulations on being awarded the National Medal of Science. I know that this comes after various other honours, including of course, the Abel Prize, so there might be an element of having got used to such events. Still, what were your feelings when you heard the news and when you actually received the medal from President Obama?

Srinivasa Varadhan: It is always gratifying to be recognised for something that one has done. I was happy for myself as well as for my family, especially for my seven-year-old grandson, Gavin, who was thrilled to attend the function and meet President Obama. It was a graceful affair and the agencies of the government that ran it did a wonderful job.

SR: The article that appeared in a leading Indian news magazine, Frontline, after you won the Abel Prize, generated a lot of interest in India (http://frontlineonnet.com/fl2407/stories/20070420001909700. htm). I would like to dwell in detail on some aspects mentioned there .... For instance, the article talks about your father and his other student V S Varadarajan .... Can you tell us a little more about your childhood, the environment at home, your schooling, etc.

SV: Varadarajan's father and my father knew each other professionally as they were both in the field of education, but V S Varadarajan was never a student of my father. We grew up in different towns. My father was a science teacher and became a headmaster at some point. He was in the district school system and was transferred periodically to different towns within Chingleput district, which surrounds the city of Chennai. I was the only child and grew up without the company of any siblings. But we had a close extended family and always visited or were visited by many cousins from both sides during vacation time. I am still close to my cousins and keep in touch with them. My schooling was always following my father around. These small towns had only one school and my

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