ON THE NATURE OF MATHEMATICS AND SCIENTIFIC KNOWLEDGE IN INDIAN TRADITION[1]

M.D.SRINIVAS
Centre for Policy Studies, Chennai
mdsrinivas50@gmail.com

Importance of Commentaries in the Study of Indian Scientific Tradition

While there have been several extensive investigations on the history and achievements of the Indian tradition of sciences, there has not been much discussion on the foundational methodology of Indian sciences. Traditionally, such issues have been dealt with in the detailed *bhāṣyas* or commentaries, which continued to be written till recent times and played a vital role in the traditional scheme of learning.

As regards Indian mathematics, it is in such commentaries that we find detailed *upapattis* or "proofs" of the results and procedures, apart from a discussion of methodological and philosophical issues. It has been the scant attention paid, by the modern scholarship of the last two centuries, to this extensive tradition of commentaries, which has led to a lack of comprehension of the methodology of Indian mathematics; and this is reflected in the often repeated statements on the absence of logical rigour in Indian mathematics in works on history of mathematics such as the following:[2]

> As our survey indicates, the Hindus were interested in and contributed to the arithmetical and computational activities of mathematics rather than to the deductive patterns. Their name for mathematics was *gaṇita*, which means 'the science of calculation'. There is much good procedure and technical facility, but no evidence that they considered proof at all. They had rules, but apparently no logical scruples. Moreover, no general methods or new viewpoints were arrived at in any area of mathematics.
>
> It is fairly certain that the Hindus did not appreciate the significance of their own contributions. The few good ideas they had, such as separate symbols for the numbers from 1 to 9, the conversion to base 10, and negative numbers, were introduced casually with no realisation that they were valuable innovations. They were not sensitive to mathematical values. Along with the ideas they themselves advanced, they accepted and incorporated the crudest ideas of the Egyptians and Babylonians.

---

[1] Revised version of M.D.Srinivas, "On the Nature of Mathematics and Scientific Knowledge in Indian Tradition," in J.M.Kanjirakkat et al. eds., *Science and Narratives of Nature East and West,* (New York: Routledge 2015), 220-238.

[2] Morris Kline, *Mathematical Thought from Ancient to Modern Times* (Oxford: Oxford University Press, 1972), 190.

It is true that *gaṇita* or Indian mathematics is quintessentially a science of computation and texts of Indian mathematics essentially present systematic and efficient procedures or algorithms for the solution of various mathematical problems. The ancient texts of geometry, *Śulbasūtras* (prior to 600 BCE), give procedures for the construction and transformation of geometrical figures. The classical text *Āryabhaṭīya* of Āryabhaṭa (c.499) presents most of the procedures of arithmetic, algebra, geometry and trigonometry, which are taught today in schools, in just thirty-two verses comprising the *Gaṇitapāda.* While the canonical texts such as the *Āryabhaṭīya* or the *Brāhmasphuṭasiddhānta* of Brahmagupta present only the results and procedures, it is the commentaries written on them which explain these results and procedures and often present detailed *upapattis* or demonstrations of them. Such commentaries formed an integral part of the traditional scheme of learning and many great authors of seminal works such as Bhāskarācārya II (c.1150), Parameśvara (c.1450) and Nīlakaṇṭha Somayājī (c. 1450-1550) also wrote important commentaries, sometimes on their own works.

In his *Vāsanābhāṣya* on his own treatise on Algebra, *Bījagaṇita*, Bhāskarācārya II (c.1150) explains that the tradition of *upapatti* has been for long a part of the oral instruction (*pāṭha-nibaddhā*).[3] The following are some of the important commentaries which are available in print and contain some discussion of *upapattis* for various results and procedures of Indian mathematics and astronomy:

1. *Bhāṣya* of Bhāskara I (c.629) on *Āryabhaṭīya* of Āryabhaṭa (c.499)
2. *Bhāṣya* of Govindasvāmin (c.800) on *Mahābhāskarīya* of Bhāskara I (c.629)
3. *Vāsanābhāṣya* of Caturveda Pṛthūdakasvāmin (c.860) on *Brāhmasphuṭasiddhānta* of Brahmagupta (c.628)
4. *Vivaraṇa* of Bhāskarācārya II (c.1150) on *Śiṣyadhīvṛddhidatantra* of Lalla (c.748),
5. *Vāsanā* of Bhāskarācārya II (c.1150) on his own *Līlāvatī, Bījagaṇita* and *Siddhāntaśiromaṇi*
6. *Siddhāntadīpikā* of Parameśvara (c.1431) on the *Bhāṣya* of Govindasvāmin (c.800) on *Mahābhāskarīya* of Bhāskara I (c.629)
7. *Āryabhaṭīyabhāṣya* of Nīlakaṇṭha Somayājī (c.1501) on *Āryabhaṭīya* of Āryabhaṭa (c.499), K. Sambasiva Sastri (ed.), 3 Vols., Trivandrum 1931, 1932, 1957
8. *Gaṇita-Yuktibhāṣā* (in Malayalam) of Jyeṣṭhadeva (c.1530)
9. *Yuktidīpikā* of Śaṅkara Vāriyār (c.1530) on *Tantrasaṅgraha* of Nīlakaṇṭha Somayājī (c.1500)
10. *Kriyākramakarī* of Śaṅkara Vāriyār (c.1535) on *Līlāvatī* of Bhāskarācārya II (c.1150)
11. *Sūryaprakāśa* of Sūryadāsa (c.1538) on *Bījagaṇita* of Bhāskarācārya II (c.1150)
12. *Buddhivilāsinī* of Gaṇeśa Daivajña (c.1545) on *Līlāvatī* of Bhāskarācārya II (c.1150)

[3] Devchandra Jha, ed., *Bījagaṇitam* (Varanasi: Chowkahmbha Prakashan, 1983), 399-400.

13. *Bījapallavam* of Kṛṣṇa Daivajña (c.1600) on *Bījagaṇita* of Bhāskarācārya II (c.1150)
14. *Vāsanāvārttika*, commentary of Nṛsiṁha Daivajña (c.1621) on *Vāsanābhāṣya* of Bhāskarācārya II on his own *Siddhāntaśiromaṇi* (c.1150)
15. *Marīci* of Munīśvara (c.1630) on *Siddhāntaśiromaṇi* of Bhāskarācārya II (c.1150)

Of these, the Malayalam text *Gaṇita-Yuktibhāṣā* of Jyeṣṭhadeva (c.1530) is a compendium which is exclusively devoted to a systematic and detailed exposition of rationales (*yuktis*) of various results and procedures in mathematics and astronomy.[4]

***Upapatti* and "Proof"**

In the introduction to the section on spherics (*Golādhyāya*) of his treatise *Siddhāntaśiromaṇi*, Bhāskarācārya II explains the central purpose behind his exposition of *upapattis*:[5]

> मध्याद्यं द्युसदां यदत्र गणितं तस्योपपत्तिं विना
> प्रौढिं प्रौढसभासु नैति गणको निःसंशयो न स्वयम् ।
> गोले सा विमला करामलकवत् प्रत्यक्षतो दृश्यते
> तस्मादस्म्युपपत्तिबोधविधये गोलप्रबन्धोद्यतः ॥
>
> Without the knowledge of *upapattis*, by merely mastering the calculations (*gaṇita*) described here, from the *madhyamādhikāra* (the first chapter of *Siddhāntaśiromaṇi*) onwards, of the [motion of the] heavenly bodies, a mathematician will not be respected in the scholarly assemblies; without the *upapattis* he himself will not be free of doubt (*niḥsaṁśaya*). Since *upapatti* is clearly perceivable in the (armillary) sphere like a berry in the hand, I therefore begin the *Golādhyāya* (section on spherics) to explain the *upapattis*.

The same is echoed by the Gaṇeśa Daivajña in his famous commentary *Buddhivilāsinī* (c.1540) on Bhāskarācārya's *Līlāvatī*:[6]

> व्यक्ते वाव्यक्तसंज्ञे यदुदितमखिलं नोपपत्तिं विना तत्
> निर्भ्रान्तो वा ऋते तां सुगणकसदसि प्रौढतां नैति चायम्।
> प्रत्यक्षं दृश्यते सा करतलकलितादर्शवत् सुप्रसन्ना
> तस्मादग्र्योपपत्तिं निगदितुमखिलम् उत्सहे बुद्धिवृद्ध्यै॥
>
> Without *upapatti*, whatever is stated in *vyakta-gaṇita* (mathematics dealing with manifest quantities – arithmetic and geometry) or *avyakta-gaṇita* (mathematics dealing with un-manifest quantities – algebra), will not be rendered free from

---

[4] K.V.Sarma et al. ed. trans., *Gaṇita-Yuktibhāṣa,* 2 Vols (Delhi: Hindustan Book Agency, 2008).
[5] Murali Dhara Chaturvedi, ed., *Siddhāntaśiromaṇi* (Varanasi: Sampurnanand Sanskrit University, 1981), 326.
[6] V.G.Apte, ed., *Līlāvatī with Buddhivilāsinī* (Pune: Nirnayasagar Press, 1937), 1.

> confusion (*nirbhrānta*); nor will it have any value in an assembly of mathematicians. *Upapatti* is indeed directly and pleasantly perceivable like a mirror in hand. It is therefore, as also for the elevation of the intellect, that I proceed to enunciate *upapattis* in entirety.

Thus, the notion of *upapatti* seems to be significantly different from the notion of "proof" as understood in the Greco-European tradition of mathematics. According to the Indian mathematical texts, the purpose of *upapatti* is mainly: (i) to remove confusion and doubts regarding the validity and interpretation of mathematical results and procedures; and, (ii) to obtain assent in the community of mathematicians. This is very different from the ideal of "proof" in the Greco-European tradition which is to irrefutably establish the absolute truth of a mathematical proposition.

Further, in the Indian tradition, mathematical knowledge is not taken to be different in any fundamental sense from that in natural sciences. In fact, valid means for acquiring and validating mathematical knowledge are the same as in other sciences: *Pratyakṣa* (perception), *Anumāna* (inference), *Śabda* or *Agama* (authentic text or tradition).

The following are some of the important features of *upapattis* in Indian mathematics:[7]

1. The Indian mathematicians are clear that results in mathematics, even those enunciated in authoritative texts, cannot be accepted as valid unless they are supported by *yukti* or *upapatti*. It is not enough that one has merely observed the validity of a result in a large number of instances.

2. Several commentaries written on major texts of Indian mathematics and astronomy present *upapattis* for the results and procedures enunciated in the text.

3. The *upapattis* are presented in a sequence proceeding systematically from known or established results to finally arrive at the result to be established.

4. In the Indian mathematical tradition the *upapattis* mainly serve to remove doubts and obtain consent for the result among the community of mathematicians.

5. The *upapattis* may involve observation or experimentation. They also depend on the prevailing understanding of the nature of the mathematical objects involved.

6. The method of *tarka* or "proof by contradiction" is used occasionally. But there are no *upapattis* which purport to establish existence of any mathematical object merely on the basis of *tarka* alone.

[7] M. D. Srinivas, "Proofs in Indian Mathematics", in G. G. Emch et al. ed., *Contributions to the History of Indian mathematics* (Delhi: Hindustan Book Agency, 2005), 231-2.

7. The Indian mathematical tradition did not subscribe to the ideal that *upapattis* should seek to provide irrefutable demonstrations establishing the absolute truth of mathematical results. There was apparently no attempt to present the *upapattis* as a part of a deductive axiomatic system. While Indian mathematics made great strides in the invention and manipulation of symbols in representing mathematical results and in facilitating mathematical processes, there was no attempt at formalization of mathematics.

Here, we may add a few remarks concerning the role of "proof by contradiction" in Indian mathematics. Indian mathematical texts do employ this method of indirect proof for proving the non-existence of an entity. For instance, we may cite the following passage from Kṛṣṇa Daivajña's commentary *Bījapallava* (c.1601) on Bhāskarācārya's *Bījagaṇita*, where he argues that negative numbers do not have any square-roots:[8]

> वर्गस्य हि मूलं लभ्यते। ऋणाङ्कस्तु न वर्गः कथमतस्तस्य मूलं लभ्यते। ननु ऋणाङ्कः कुतो वर्गो न भवति न हि राजनिर्देशः। ...सत्यम्। ऋणाङ्कं वर्गं वदता भवता कस्य स वर्ग इति वक्तव्यम्। न तावद्धनाङ्कस्य 'समद्विघातो हि वर्गः' तत्र धनाङ्केन धनाङ्के गुणिते यो वर्गो भवेत् स धनमेव 'स्वयोर्वधः स्वम्' इत्युक्तत्वात्। नाप्यृणाङ्कस्य। तत्रापि समद्विघातार्थमृणाङ्केनर्णाङ्कगुणिते धनमेव वर्गो भवेत् 'अस्वयोर्वधः स्वम्' इत्युक्तत्वात्। एवं सति कथमपि तमङ्कं न पश्यामो यस्य वर्गः क्षयो भवेत्।
>
> The square-root can be obtained only for a square. A negative number is not a square. Hence how can we consider its square-root? It might however be argued: 'Why will a negative number not be a square? Surely it is not a royal fiat.'... Agreed. Let it be stated by you who claim that a negative number is a square as to whose square it is. Surely not of a positive number, for the square of a positive number is always positive by the rule ... Not also of a negative number. Because then also the square will be positive by the rule... This being the case, we do not see any such number whose square becomes negative.

However, there are no instances where Indian mathematicians use the method of indirect proof to establish the existence of an entity, the existence of which is not demonstrable (even in principle) by other (direct) means of verification. In this sense, the Indian mathematical tradition may be seen as adopting what is nowadays referred to as the "constructivist" approach to the issue of mathematical existence.

It is important to note that this significant feature of Indian mathematical tradition is closely related to the world-view of the Naiyāyikas or Indian logicians, who do not accord *tarka* (or the method of indirect proof) the status of an independent source of valid knowledge (*pramāṇa*). Indeed the general philosophical approach of Indian logicians is one of eliminating from logical discourse all reference to such *aprasiddha* or un-instantiated

---

[8] T.V.Radhakrishna Sastry, ed., *Bījagaṇitam with Bījapallavam* (Tanjore: Saraswati Mahal Library, 1958), 19.

entities, whose existence is not even in principle accessible to direct means of verification. In fact, the Naiyāyikas would even reconcile to live with contradictions rather than allow the use of such *aprasiddha* entities in their logical discourse. This is brought out very clearly by Matilal by citing an important passage from *Ātmatattvaviveka* of Udayanācārya (c. 10th century) which deals with an argument between the Bauddhas and Naiyāyikas:[9]

> Nyāya...[excludes] from logical discourses any sentence which will ascribe some property (positive or negative) to a fictitious entity. Vācaspati remarks that we can neither affirm nor deny anything of a fictitious entity, the rabbit's horn. Thus Nyāya apparently agrees to settle for a superficial self-contradiction because, in formulating the principle that nothing can be affirmed or denied of a fictitious entity like rabbit's horn, Nyāya, in fact violates the same principle. Nyaya feels that this superficial self-contradiction is less objectionable [than admitting fictitious entities in logical discourse]... By way of documentation...[is given] below the translation of an excerpt from Udayana's *Ātmatattvaviveka*...[10]
>
> '(Proponent:) ... There are some other defects in this negative inference. The minor term (the "subject" *pakṣa*), the middle term (the "inferential reason" *hetu*) and the example cited in such an inference cannot be established by any means of knowledge. There cannot be any means of knowledge to establish a non-entity (i.e., a fiction, *avastu*). If it could be established by some means of knowledge, it ceases to be non-entity.
>
> '(Opponent:) If so, then your talk about the non-entity becomes self-contradictory.
>
> '(Proponent:) Does this self-contradiction point out that there is a means of knowledge to establish the non-entity? Or, (second question) does it reject the prohibitive statement that we should not talk about non-entity? Or (third question)

---

[9] Bimal Krishna Matilal, *Logic, Language and Reality* (Delhi: Motilal Banarsidass, 1985), 103-4.

[10] The original text is reproduced below:

तत्राश्रयहेतुदृष्टान्तसिद्धौ प्रमाणाभाव: अवस्तुनि प्रमाणाप्रवृत्तेः प्रमाणप्रवृत्तौ अलीकत्वानुपपत्ते:।

एवं तर्ह्यव्यवहारे स्ववचनविरोधः स्यादिति चेत्।

तत्किं स्ववचनविरोधेन तेषु प्रमाणमुपदर्शितं भवेत् व्यवहारनिषेधव्यवहारो वा खण्डितः स्यात् अप्रमाणिकोऽयं व्यवहारो अवश्याभ्युपगन्तव्य इति वा भवेत्। न तावत्प्रथम:। न हि विरोधसहस्रेणापि स्थिरे तस्य क्रमादिविरहे वा शशशृङ्गे वा प्रत्यक्षमनुमानं वा दर्शयितुं शक्यम्। तथात्वे वा कृतं भौतकलहेन। द्वितीयस्त्विष्यत एव प्रामाणिकैः।

अवचनम् एव तर्हि तत्र प्राप्तम् ।

किं कुर्मो यत्र वचनं सर्वथैवानुपपन्नं तत्रावचनमेव श्रेयः। त्वमपि परिभावय तावन्निष्प्रामाणिकेऽर्थे मूकवावदूकयोः कतरः श्रेयान्।

एवं विदुषापि भवता न मूकीभूय स्थितमपि तु व्यवहारः प्रतिषिद्ध एवासतीति चेत्।

सत्यम्। यथाऽप्रामाणिकः स्ववचनविरुद्धोऽर्थो मा प्रसङ्क्षीदिति मन्यमानेन त्वया चाप्रामाणिक एवासति व्यवहारः स्वीकृतः तथास्माभिरपि प्रमाणचिन्तायामप्रामाणिको व्यवहारो मा प्रसङ्क्षीदिति मन्यमानैरप्रामाणिक एव स्ववचनविरोधः स्वीक्रियते। यदि तूभयत्रापि भवान्समानदृष्टिः स्यादस्माभिरपि तदा न किंचिदुच्यत इति ।

does it imply that we must concede such statements (about non-entity), which are unauthenticated, i.e., not established by any means of knowledge? The first alternative is not tenable. Even a thousand of self-contradictions cannot conceivably show that (the non-entity like) the stable object (i.e., the minor term) or the absence of gradual efficiency, etc. (i.e., the *hetu*) or the rabbits' horn (i.e., the example cited to support the general premise) is amenable to (a means of knowledge, such as) perception and inference. If it could, what is the use of this silly fight over the nature of non-entities? The second alternative is acceptable to us, because we admit only valid means of knowledge.

'(Opponent:) If the prohibitive statement is rejected, no statement with regard to non-entities will be possible.

'(Proponent:)What else can we do but remain silent in regard to a matter where statement of any kind will be logically incongruent? Silence is better in such cases. (No statement is better than any statement in such matters.) You yourself may please consider as to who is the better of the two: One who is making statements about entities that cannot be established by any means of knowledge? Or, the other person who remains speechless (on such occasions)?

'(Opponent:) But although you are a wise man you have not remained silent yourself. You on the other hand have made a prohibitive statement with regard to our talk about non-entities.

'(Proponent:) True, in order to avoid a self-contradictory object not established by any means of knowledge, you have conceded that one can make statements about the non-existent. Similarly, in order not to allow any statement about the non-entities in our discourse on the means of knowledge, we concede that a self-contradictory statement (prohibiting the use of non-entities) is possible, although it is not supported by any means of knowledge. If you treated both the cases in the same manner, we would not have said anything about non-entities. (We have made the above self-contradictory statement because you first raised the question).'

Indian Grammarians' View of Śāstra as Upāya

To understand the methodology of Indian sciences, one has to perhaps start with the foundational works on Indian linguistics, not only because linguistics is the earliest of Indian sciences to have been rigorously systematised, but also because this systematisation became the paradigm example for all other sciences. It has been aptly remarked that the *Aṣṭādhyāyī* of Pāṇini (prior to 500 BCE) enjoys the same kind of prestige in Indian tradition as the *Elements* of Euclid does in Greco-European tradition:[11]

[11] J. F. Staal, "Euclid and Pāṇini", *Philosophy East and West* 15 (1965): 113-4. It should however be noted that the word 'derived' in this passage refers to derivation in the sense of logical demonstration in the case

> In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed ad hoc (i.e. *sūtras*)...
>
> Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in western thought. Scientific developments have therefore taken different directions in India and in the West.... In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. In the west this corresponds to the belief that mathematics is the more perfect among the sciences. Just as Plato reserved admission to his Academy for geometricians, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics. In India, grammar was called the Veda of the Vedas, the science of sciences.

It is now generally appreciated that the *Aṣṭādhyāyī* of Pāṇini, gives a systematic way of generating all the valid utterances of Sanskrit, in terms of about 4000 grammatical rules supplemented by an inventory of about 2000 verbal bases (*Dhātupāṭha*) and some 261 lists of lexical bases (*Gaṇapāṭha*). In his famous commentary *Mahābhāṣya*, on Pāṇini's *Aṣṭādhyāyī*, Patañjali (c.2nd Century BCE) explains that the purpose of grammar is to give an exposition of all valid utterances. An obvious way to do this is to enumerate all valid utterances individually. Since that is humanly impossible, one should attempt to encapsulate larger and larger class of valid utterances by means of a set of general (*utsarga*) and exceptional (*apavāda*) rules. Patañjali further emphasises that the utterances and their meanings are actually established in the world – one does not go to a Grammarian to make utterances for him as one goes to a potter for pots.[12]

In thus characterising Pāṇinian grammar, Patañjali expounds what is perhaps the basic understanding of the Indian scientific effort:[13]

> Science in India seems to start with the assumption that truth resides in the real world with all its diversity and complexity. For the linguist, what is ultimately true is the language as spoken by the people in all their diverse expressions... Linguists do make generalisations about the language as spoken in the world. But these generalisations are not the truth behind or above the reality. These are not the

---

of Euclid's geometry, but it refers to derivation in the sense of generation of linguistic forms from grammatical elements in the case of Pāṇini's grammar.

[12] See for instance, S. D. Joshi and J.A.F. Roodbergen, trans., *Vyākaraṇa Mahābhāṣya Paspaṣāhnika* (Pune: Motilal Banarsidass, 1986), 16-25, 70-117.

[13] J. K. Bajaj, "The Indian Tradition of Science and Technology An Overview," *PPST Bulletin*, 13-14 (March 1988): 33. See also J.K.Bajaj, "Science and Technology Up to 1800", in F.Robinson ed., *Cambridge Encyclopedia of India, Pakistan and Bangladesh* (Cambridge: Universities Press, 1989), 496-7.

idealisation according to which reality is to be tailored. On the other hand what is ideal is the real, and some part of the real always escapes our idealisation of it. There are always exceptions. It is the business of the scientist to formulate these generalisations, but also at the same time to be always attuned to the reality, to always be conscious of the exceptional nature of each specific instance. This attitude seems to permeate all Indian science and makes it an exercise quite different from the scientific enterprise of the west.

Many of these issues discussed by Patañjali are further investigated by the great philosopher Bhartṛhari (c.500 CE) in his treatise *Vākyapadīya*. Texts of Indian astronomy often cite his famous dictum that the procedures taught in *śāstras* are only means (*upāya*) to accomplish desired objectives in the world and they are not constrained or regulated in any other manner:[14]

उपादायापि ये हेया तानुपायान् प्रचक्षते।
उपायानाञ्च नियमो नावश्यमवतिष्ठते ॥
अर्थं कथञ्चिद् पुरुष: कथञ्चित्प्रतिपद्यते।

Upāyas (procedures taught in *śāstras*) are to be discarded, even though they are to be used for accomplishing an objective. There is no necessary limitation on such *upāyas*. One accomplishes objectives by one means or the other.

In this context Puṇyarāja, the commentator on *Vākyapadīya*, notes:

शास्त्रमुपाय: शब्दपरिज्ञाने। ज्ञातेषु तेषु प्रयोजनसम्पत्तेरनुपयोगः इति तस्य परित्यागः। उपायाश्च न नियता इत्याह ।...कश्चिदाचार्यः पाणिनिविरचितेन लक्षणशास्त्रेण शब्दानधिगच्छति कश्चिदन्येनेति न नियम:।

The science of grammar is a means for knowing the meanings of utterances. Once these are known there is no further use and hence it is said that they are to be discarded. He also states that there is no limitation on these *upāyas*.... One preceptor (Ācārya) understands utterances by means of the grammatical framework of Pāṇini and another by means of another framework and thus there is no rule [that only a particular grammar is to be followed].

This pragmatic approach to scientific theorisation indeed becomes folklore as it were in Indian philosophical thought. The great eighteenth century scholar Nāgeśabhaṭṭa begins his treatise on the philosophy of grammar, *Paramalaghumañjūṣā,* with a reiteration of

[14] Raghunatha Sharma, ed., *Vākyapadīyam Vākyakhaṇḍam with Puṇyarāja Commentary* (Varanasi: Sampurnanand Sanskrit University, 1980), 79-81.

this point that grammatical derivations are *upāyas* and are otherwise unrestricted (*avyavasthita*):[15]

> तत्र वाक्यस्फोटो मुख्यः तस्यैवलोकेऽर्थबोधकत्वात्तेनैवार्थसमाप्तेश्चेति। ...तत्र प्रतिवाक्यं सङ्केतग्रहासम्भवाद् वाक्यान्वाख्यानस्य लघूपायेनाशक्यत्वाच्च कल्पनया पदानि प्रविभज्य पदे प्रकृतिप्रत्ययविभागान्प्रविभज्य कल्पिताभ्यामन्वयव्यतिरेकाभ्यां तत्तदर्थविभागं शास्त्रमात्रविषयं परिकल्पयन्ति स्माचार्याः।...
>
> मुख्यं वाचकत्वं तु कल्पनया बोधितसमुदायरूपे पदे वाक्ये वा लोकानां तत एवार्थबोधात्। 'उपेयप्रतिपत्त्यर्था उपाया अव्यवस्थिता' इति न्यायेन व्याकरणभेदेन स्थानिभेदेऽपि न क्षतिः देशभेदेन लिपिभेदवदिति दिक्।
>
> There (amongst the syllable, word and sentence meanings), it is the sentential meaning (*vākyasphoṭa*) that is the primary; for it is the sentence which is seen to have import and completeness of meaning in the world....Since it is not feasible to identify all the (valid) sentences, and (mere consideration of sentences) will not provide any simple means for explaining sentence-meaning, the preceptors (Ācāryas) have devised a fictitious procedure, wherein sentences are divided into words and words into stems (*prakṛti*) and suffixes (*pratyaya*) and, following the procedure of mutual presence and absence (*anvayavyatireka*), they conceive of imputed meanings for these units only for the purpose of grammatical derivation (*śāstra*)...
>
> Meaningfulness (*vācakatva*) rests mainly in the words or sentences which are made up of these imagined entities; for, in the world, only these (words and sentences) convey meanings. Indeed, following the well known principle that 'the *upāyas* (grammatical derivations) are only for the realisation of the desired result and are otherwise unrestricted (*avyavasthita*),' there should be no cause of concern even if different substituends are employed in different grammars. It should be noted that this is akin to the fact that the script may change with a change in locality.

Status of Planetary Models in Indian Astronomy

The tradition of astronomy in India goes back to the ancient texts of *Vedāṅgajyotiṣa* which give simple algorithms for fixing the elements of Indian calendar (*Pañcāṅga*). The *Vedāṅgajyotiṣa* texts, as well as the later elaborate treatises on Indian astronomy, declare the *raison d'être* of the science of astronomy to be the determination of time (*kālavidhānaśāstra*), as well as position and direction, by means of the motion of the celestial bodies. Hence, it is the pragmatic concerns of calculating the positions of the various planets and eclipses of the Sun and the Moon reasonably accurately, which informed the efforts of the Indian Astronomers and in this they seem to have been eminently successful at least from the time of Āryabhaṭa.

---

[15] Alakhdeva Sarma, ed. & trans., *Paramalaghumañjūṣā* (Varanasi: Chowkhambha Prakashan, 1981), 4-5.

Though there are several references to earlier *siddhāntas*, the earliest available systematic exposition of planetary theory is contained in the *Āryabhaṭīya* (c.499 CE) of Āryabhaṭa and in the works of his commentator Bhāskara I (c.629). Here, the procedure for calculating the geocentric longitudes of the planets, Mercury, Venus, Mars, Jupiter and Saturn involves essentially the following steps.[16] First, the mean longitude is calculated for the desired day and then two corrections, namely the *manda-saṁskāra* and *śīghra-saṁskāra,* are applied to the mean planet to obtain the true longitude. In the case of the exterior planets, Mars, Jupiter and Saturn, the *manda-saṁskāra* is equivalent to taking into account the eccentricity of the planet's orbit around the Sun and the *manda* correction coincides, to first order in eccentricity, with the equation of centre currently calculated in astronomy. This is followed by the *śīghra- saṁskāra,* which is equivalent to converting the heliocentric longitude into the geocentric longitude.

While explaining the planetary model as expounded by Āryabhaṭa, Bhāskara I notes that notions such as the apsides (*ucca, nīca*), mean (*madhyama*), epicycles (*paridhi*) etc., are conceptual tools which serve the purpose of arriving at the observed motion of planets and there are no constraints on them except that they should lead to observed results:[17]

> उच्चनीचमध्यमपरिधिरित्येवमादिस्फुटगतिसाधनोपाय [भूतानाञ्च] उपायानां नैव नियमोक्तिर्वा विद्यते। केवलं तु उपेयसाधका उपायाः। तस्मादियं सर्वा प्रक्रिया असत्या यया ग्रहाणां स्फुटगतिः साध्यते। एवं च परमार्थजिज्ञासुभिः असत्योपायेन सत्यं प्रतिपद्यते। तथा हि भिषजो ह्युत्पलनालादिषु वेधादीन्यभ्यस्यन्ते नापिता: पिठरादिषु मुण्डनादीनि यज्ञशास्त्रविदः शुष्केष्ट्या यज्ञादीनि शाब्दिकाः प्रकृतिप्रत्ययविकारागमवर्णलोपव्यत्ययादिभिः शब्दान् प्रतिजानते। एवमत्रापि मध्यममन्दोच्चशीघ्रोच्चतत्परिधिज्याकाष्ठभुजाकोटिकर्णादिव्यवहारेण सांवत्सरा ग्रहाणां स्फुटगतिं प्रतिजानते। तस्मादुपायेष्वसत्येषु सत्यप्रतिपादनपरेषु न चोद्यमस्ति।
>
> There are no constraints or limitations imposed on the notions such as *ucca*, *nīca*, *madhyama, paridhi* and so on, which are essentially aids to the calculation of the observed motion of the planets. They are only the means for arriving at the desired results. Hence this entire procedure is fictitious, by means of which the observed planetary motion is arrived at....Just as the linguists utilise notions such as *prakṛti*, *pratyaya, vikāra, āgama, varṇa, lopa, vyatyaya*, etc., to comprehend (well-formed) words.... In the same way in our science also astronomers employ notions such as *madhyama, mandocca, śīghrocca, śīghraparidhi, jyā, kāṣṭha, bhujā*, *koṭi, karṇa*, etc., in order to comprehend the observed motion of planets. Hence, there is indeed

[16] For an overview of the development of Indian planetary theories, see K. Ramasubramanian and M. S. Sriram, trans., *Tantrasaṅgraha of Nīlakaṇṭha Somayājī* (New York: Springer Verlag, 2011), 487-535.

[17] Kripa Shankar Shukla, ed., *Āryabhaṭīya with Bhāṣya of Bhāskara* (New Delhi, Indian National Science Academy, 1976), 217.

> nothing unusual that fictitious means are employed to arrive at the true state of affairs [in all these *śāstras*].

Thus, the Indian astronomers were in the business to calculate and to compute, not to form pictures of the heavens as they ought to be. Indian astronomers do employ various models, analytical as well as geometrical, but (as we have seen above) the texts themselves emphasise, these are no more than artefacts used in their calculations.

In their attempt to achieve concordance between their calculations and the observed planetary motions, Indian astronomers were sometimes ready to accommodate inexplicable or even seemingly contradictory procedures as component part of their models. In the traditional planetary model of Āryabhaṭa, in the case of Mercury and Venus, the mean Sun is taken as the mean planet and the equation of centre is applied to it. This was indeed a feature common to all the ancient planetary theories (Indian, Greco-European & Islamic). However, the traditional Indian planetary model managed to achieve a far more accurate description of the planetary latitudes (than was achieved in the Greco-European and Islamic traditions in the pre-modern period) by employing the notion of *śīghrocca* which, in the case of the interior planets Mercury and Venus, corresponds to the mean heliocentric planet. Āryabhaṭa's prescription was that the latitudinal motion of the interior planet is to be found from its *śīghrocca*. Brahmagupta went on to suggest that one should actually employ the *manda*-corrected *śīghrocca* and in this way he was able to ensure that the latitude is calculated from the true heliocentric longitude of the planet.

Thus, we see that the traditional Indian texts did provide a fairly accurate theory of the planetary latitudes. But, in the process, they had to live with two entirely different rules for calculating latitudes, one for the exterior planets where the *manda*-corrected mean longitude appears and an entirely different one for the interior planets which involves the *manda*-corrected *śīghrocca* of the planet. This peculiarity of the rule for calculating the latitude of an interior planet was repeatedly noticed by various Indian astronomers, at least from the time of Bhāskara I (c.629) onwards. The celebrated astronomer Bhāskarācārya II (c.1150) also draws attention to this peculiar procedure adopted for the interior planets, in his *Vāsanābhāṣya* on his own *Siddhāntaśiromaṇi*, and quotes the statement of Caturveda Pṛthūdakasvāmin (c.860) that this peculiar procedure for the interior planets can be justified only on the ground that this is what has been found to lead to predictions that are in conformity with observations:[18]

> ननु ज्ञशुक्रयोः शीघ्रोच्चपातयुतिं केन्द्रं कृत्वा यो विक्षेप आनीतः स शीघ्रोच्चस्थान एव भवितुमर्हति। न ग्रहस्थाने। यतो ग्रहोऽन्यत्र वर्तते। अत इदमनुपपन्नमिव प्रतिभाति। तथा च ब्रह्मसिद्धान्तभाष्ये। ज्ञशुक्रयोः शीघ्रोच्चस्थाने यावान् विक्षेपस्तावानेव यत्र तत्रस्थस्यापि ग्रहस्य भवति।अत्रोपलब्धिरेव वासना नान्यत्कारणं वक्तुं शक्यत इति चतुर्वेदेनाप्यध्यवसायोऽत्र कृतः।

[18] Murali Dhara Chaturvedi, *Siddhāntaśiromaṇi,* 402.

> The latitude (deflection from the ecliptic) that is obtained by using the *śīghrocca* and the node must be the latitude at the location of *śīghrocca* and not at the location of the planet, as the planet is somewhere else. Therefore this (procedure for the computation of latitudes of an interior planet) seems to be without any justification. However even Caturvedācārya (Pṛthūdakasvāmin) has concluded as follows in his commentary on *Brāhmasphuṭasiddhānta*: 'The latitude at the location of the *śīghrocca* of the planets Mercury and Venus, corresponds to the latitude of the planet itself wherever it may be; here the *upalabdhi* (agreement between the calculated results and observations) is the only justification as we are unable to give any other reason.'

In fact, in an attempt to resolve this seeming contradiction in the traditional method of calculation of the latitudes of the planets, the celebrated Kerala astronomer Nīlakaṇṭha Somayājī (c.1444-1550) came up with a fundamental revision of the traditional planetary theory. In his treatise *Tantrasaṅgraha* (c.1500), Nīlakaṇṭha proposed that what was till then thought of as the *śīghroccas* of Mercury or Venus should be identified with the (mean) planets themselves. This led to a more accurate formulation of the equation of centre and the latitude of the interior planets than was available either in the earlier Indian works or in the Greco-European or the Islamic traditions of astronomy till the work of Kepler, which was to come more than a hundred years later. (Incidentally, it may be noted that the celebrated works of Copernicus (c.1543) or Tycho Brahe (c.1583) did not bring about any improvement in the planetary theory of interior planets as they merely reformulated the ancient planetary model of Ptolemy for different frames of reference). In fact, in so far as the computation of the planetary longitudes and latitudes is concerned, Nīlakaṇṭha's revised planetary model closely approximates the Keplerian model, except that Nīlakaṇṭha conceives of the planets as going in eccentric orbits around the mean Sun rather than the true Sun.

In his *Āryabhaṭīyabhāṣya,* Nīlakaṇṭha has presented the detailed rationale for his revision of the traditional planetary theory:[19]

> शीघ्रवशाच्च विक्षेप उक्तः। कथमेतद्युज्यते। ननु स्वबिम्बस्य विक्षेपः स्वभ्रमणवशादेव भवितुमर्हति। न पुनरन्यभ्रमणवशादिति। सत्यम्। न पुनरन्यस्य भ्रमणवशादन्यस्य विक्षेप उपपद्यते। तस्मात् बुधोऽष्टाशीत्यैव दिनैः स्वभ्रमणवृत्तं पूरयति।... एतच्च नोपपद्यते यदेकेनैव संवत्सरेण तत्परिभ्रमणमुपलभ्यते नैवाष्टाशीत्या दिनैः। सत्यम् भगोलपरिभ्रमणं तस्यप्येकेनैवाब्देन। ...
>
> एतदुक्तं भवति। तयोर्भ्रमणवृत्तेन न भूः कबलीक्रियते। ततो बहिरेव सदा भूः। भगोलैकपार्श्व एव तद्वृत्तस्य परिसमाप्तत्वात् तद्भगणेन न द्वादशराशिषु चारः स्यात्। तयोरपि वस्तुत आदित्यमध्यम एव शीघ्रोच्चम्। शीघ्रोच्चभगणत्वेन पठिता एव स्वभगणाः। तथाप्यादित्य--

[19] S.K.Pillai, ed., *Āryabhaṭīya Golapāda with Bhāṣya of Nīlakaṇṭha* (Trivandrum:Trivandrum Sanskrit Series, 1957), 8-9.

> भ्रमणवशादेव द्वादशराशिषु चारः स्यात्। शीघ्रवृत्तस्य कक्ष्यायाः महत्त्वात्। शीघ्रोच्चनीच-वृत्तस्याप्येकभागगमेव स्वभ्रमणवृत्तम्। यथा कुजादीनामपि शीघ्रोच्चं स्वमन्दकक्ष्या-मण्डलादिकमाकर्षति एवमेतयोरपि। अनयोः पुनस्तदाकर्षणवशादेवद्वादशराशिषु चारः इति।
>
> The latitudinal motion is said to be due to that of the *śīghrocca*. How is this appropriate? Isn't the latitudinal motion of a body dependent on the motion of that body only, and not because of the motion of something else? The latitudinal motion of one body cannot be obtained as being due to the motion of another. Hence [we should conclude that] Mercury goes around its own orbit in 88 days... However, this also is not appropriate because we see it going around [the Earth] in one year and not in 88 days. True, the period in which Mercury completes one full revolution around the *bhagola* (the celestial sphere) is one year only [like the Sun]...
>
> All this can be explained thus: Their [Mercury and Venus] orbits do not circumscribe the earth. The Earth is always outside their orbit. Since their orbit is always confined to one side of the [geocentric] celestial sphere, in completing one revolution they do not go around the twelve signs (*rāśis*). For them also really the mean Sun is the *śīghrocca*. It is only their own revolutions, which are stated to be the revolutions of the *śīghrocca* [in the *Āryabhaṭīya*]. It is only due to the revolution of the Sun [around the Earth] that they (i.e. the interior planets, Mercury and Venus) complete their movement around the twelve signs [and complete their revolution of the Earth]. Because the *śīghra*-epicycle is larger than their orbit, their orbit is completed on one side of the *śīghra*-epicycle. Just as in the case of the Jupiter etc. [the exterior planets] the *śīghrocca* (i.e., the mean Sun) attracts [and drags around] the *manda*-orbits on which they move (*manda-kakṣyā-maṇḍala*), in the same way it does in the case of these [interior] planets also. And it is due to this attraction that these [interior planets] move around the twelve signs.

The above passage also exhibits the clinching argument employed by Nīlakaṇṭha. Starting from the fact that the motion of the interior planets was characterised by two different periods, one for their latitudinal motion and another for their motion in longitude, Nīlakaṇṭha arrived at what may be termed a revolutionary discovery concerning the motion of the interior planets: That they go around the Sun in orbits that do not circumscribe the Earth in a period that corresponds to the period of their latitudinal motion and that they go around the Zodiac in one year as they are dragged around the Earth by the Sun.

It was indeed well known to the ancients that the exterior planets, Mars, Jupiter and Saturn, go around the Earth and that they also go around the Sun in the same mean period, because their geocentric orbit was outside that of the Sun. Nīlakaṇṭha was the first savant in the history of astronomy to clearly derive from the computational scheme, and not from any speculative or cosmological argument, that the interior planets go around the Sun in an orbit that does not enclose the Earth, and the period of their motion around Sun is also the period of their latitudinal motion.

In his works, *Golasāra*, *Siddhāntadarpaṇa*, and a short but remarkable tract *Grahasphuṭānayane Vikṣepavāsanā*, Nīlakaṇṭha describes the geometrical picture of planetary motion that follows from his revised planetary theory, according to which the five planets Mercury, Venus, Mars, Jupiter and Saturn move in eccentric orbits (inclined to the ecliptic) around the mean Sun, which in turn goes around the Earth. (This geometrical picture is the same as the model of solar system proposed in 1583 CE by Tycho Brahe, albeit on entirely different considerations). Most of the Kerala astronomers who succeeded Nīlakaṇṭha, such as Jyeṣṭhadeva, Acyuta Piśāraṭi, Putumana Somayājī, etc., seem to have adopted his revised planetary model.

In his other great work, *Jyotirmīmāṁsā*, Nīlakaṇṭha has highlighted the importance of preparing the practitioners of the science of astronomy for the onerous task of continuously observing the skies, continuously checking their computations against observations and repeatedly re-adjusting their parameters and theoretical procedures so as to make their calculations accord with reality. Indian astronomers had always been acutely aware that their astronomical parameters and even theoretical procedures could get out of tune with reality sooner or later, and the Indian texts repeatedly emphasise the need for updating and revising the parameters and theoretical schemes so that their computations conform to observations. In *Jyotirmīmāṁsā,* Nīlakaṇṭha has dealt with this issue in great detail as is evident from the following somewhat long quotation from this seminal work:[20]

> नन्वेवमपि स्वकाल एव गीतिकोक्तभगणाद्याः [सूक्ष्माः यदा] गीतस्य ग्रहणस्य च प्रत्यक्षसंवादः स्यात् यत इदानीं ग्रन्थकरणकालात् तृतीये दिव्याब्दे महान् भेद उपलभ्यते। गीतिकोक्तकालतः पश्चादेव हीदानीं सर्वाण्यपि ग्रहणानि दृश्यन्ते।...
>
> एवमादिदूषणं परैरुद्भाव्यमानं परिहर्तुं परीक्षाप्रकारमाह यदर्थं पदत्रयेण सकला युक्तयः प्रदर्शिताः 'क्षितिरवियोगाद् दिनकृद् रवीन्दुयोगाद्' इति। अत्रोक्ताभिर्युक्तिभिरेव बुद्धिमद्भिः सम्यक् परीक्षणं शक्यं कर्तुम्।
>
> ननु तपोभिः प्रसन्नो ब्रह्मा आर्यभटाय भगणपरिध्यादिकं ग्रहगणनसाधनभूतं संख्याविशेषमुपदिदेश। तदुपदिष्टं पुनरार्यभटः सर्वं यथोपदिष्टमेव दशभिर्गीतिभिः निबबन्ध इति केचिन्मन्यन्ते। तस्य कुतः परीक्षणं ब्रह्मणः सर्वज्ञत्वात् रागद्वेषाद्यभावाच्च अविततत्त्वनिश्चयात् इति चेत् मन्द मैवम्। देवताप्रसादो मतिवैमल्यहेतुरेव। न च पुनः ब्रह्मा आदित्यो वा स्वयमेवागत्य

[20] K.V.Sarma, ed., *Jyotirmīmāṁsā of Nīlakaṇṭha* (Hoshiarpur, Punjab University Indological Series, 1977), 1-8.

उपदिशेत्। एवमेव च वक्ष्यति चानन्तरसूत्रे 'सदसज्ज्ञानसमुद्रात् समुद्धृतं देवताप्रसादेन। सज्ज्ञानोत्तमरत्नं मया निमग्नं स्वमतिना वा॥' इति ...

'ज्योतिश्शास्त्रे [ऽपि युगपरिवृत्तिपरिमाण] द्वारेण चन्द्रादित्यादिगतिविभागेन तिथिनक्षत्र-ज्ञानमविच्छिन्नसंप्रदायगणितानुमानमूलम्'इति वार्त्तिककारोऽपि ग्रहगतिज्ञानमनुमानेनाह ।

तत्राविच्छिन्नसंप्र[दायपदमप्ये]वं व्याचष्टे। 'गणितोन्नीतस्य चन्द्रादेः देशविशेषान्वयस्य प्रत्यक्षेण संवादः ततो निश्चितान्वयस्य परस्य गणितलिङ्गोपदेशः ततस्तस्याप्तोपदेशावगतान्वयस्य अनुमानं संवादः परस्मै चोपदेशः इति सम्प्रदायाविच्छेदात् प्रामाण्यम्' इति । ...

तस्मात् शिष्यप्रशिष्यपरंपरया सर्वैरपि परीक्षणं कार्यम् ।...

आर्यभटीयस्य च परीक्षापरत्वादेव सकलदेशकालयोः स्फुटार्थत्वं न पुनः तदुक्तभगणादि-वैशिष्ट्यात् । अत इदमेव परीक्षासूत्रं सिद्धान्तान्तरेभ्योऽस्य गौरवमापादयति ।

मानसव्याख्यातापि कश्चिदाह 'ननु पैतामहादिभेदेन परस्परविरुद्धाश्च सिद्धान्ता भवन्ति। सिद्धान्तभेदे सति कालभेदः। कालभेदे सति कालाङ्गानि श्रौतस्मार्तलौकिकानि कर्माणि विफलानि स्युः। कर्मवैकल्ये सति लोकयात्रोच्छेदः। हा धिक् सङ्कटे महति पतिताः स्मः।'

अत्रोच्यते 'ऋजुमते स न शोचितव्यः। गुरुचरणपरिचरणपरैः किमिव न ज्ञायते। पञ्चसिद्धान्तास्तावत् क्वचित्काले प्रमाणमेव इत्यवगन्तव्यम्। अपि च यः सिद्धान्तः दर्शनविसंवादी भवति सोऽन्वेषणीयः। दर्शनसंवादश्च तदानीन्तनैः परीक्षकैर्ग्रहणादौ विज्ञातव्यः। ये पुनरन्यथा प्राक्तनसिद्धान्तस्य भेदे सति यन्त्रैः परीक्ष्य ग्रहाणां भगणादि ज्ञात्वा अभिनवसिद्धान्तः प्रणेय इत्यर्थात् तत् त इहलोकेऽहसनीयाः परलोकेऽदण्डनीयाश्च' इति ।...

तस्मात् शिष्याणां ग्रहगतिसामर्थ्यापादनमेव शास्त्रप्रयोजनम्। ते पुनः दृक्संवादिकरणं कृत्वा लोके सञ्चरेयुः। करणनामेव हि व्यावहारिकत्वं सूक्ष्मत्वं च स्यात्।

The number of revolutions etc., enunciated in the *Gītikā* [*pāda* of *Āryabhaṭīya*] are accurate only at the time of its composition, when they would have been tested for consonance with eclipses etc. Currently, in the third divine year [of 360 years each] after the composition of the text, one finds great differences [between calculations and observations]. All eclipses are now seen at times later than those computed [according to *Āryabhaṭīya*]....

It is only to lay at rest such criticism, which is bound to be made by others, that [Āryabhaṭa] gave the method of examination (*parīkṣāprakāra*), all the techniques of which are expressed merely by the three words 'The Sun [is ascertained] by the conjunction of the earth and Sun, by the conjunction of the Sun and the Moon [the Moon is ascertained].' By following these techniques only, the wise can do proper examination.

Some people indeed believe that, pleased by his penance (*tapas*), Brahmā instructed Āryabhaṭa the number of revolutions, [dimensions of] epicycles etc., which are to be employed in calculating the motion of planets; and that Āryabhaṭa encapsulated all that instruction faithfully in ten *Gītikā* verses. And so, [you may argue], how can we conceive of putting that [instruction] to test, since Brahmā is indeed omniscient and free from all mental aberrations such as attachment, hatred etc., and is certainly free of error? You dim-witted, it is not so. The grace of gods is only for attaining clarity of intellect. Again it cannot be that Brahma or Sun would come himself and instruct. In fact [Āryabhaṭa] states more or less the same in a later verse.

'By the grace of Brahmā, the precious jewel of excellent knowledge [of *Jyotiṣa*] has been brought out by me by means of the ship of my intellect from the sea of true and false knowledge, by diving deep into it.'...

The author of *Tantravārttika* [Kumārilabhaṭṭa] also has stated that the knowledge of the motion of planets is through inference, by noting that 'in astronomy also the knowledge of *tithi* and *nakṣatra* is founded on an unbroken tradition of calculation and inference, based on the measure of *yugas*, and the rates of motions of the Sun, Moon etc.' The 'unbroken tradition' is also explained [in the commentary *Ajitā* of Paritoṣamiśra] as follows: 'The correlation of the computed Moon etc., with actual observation at a particular place, the inferred revised computation on the basis of such correlation being transmitted as tradition, it being correlated again (with observation and again revised) and transmitted further down to others ... this is how tradition is continued without interruption, and hence its continued validity.'

Therefore *parīkṣā* (examination) is to be done continuously, following the tradition of disciples and their disciples etc., by all...

It is only because *Āryabhaṭīya* has enunciated the supremacy of *parīkṣā*, that it is a relevant and valid text for all places and times, and not because of any specialty of the revolution numbers and other parameters stated therein. It is this rule of *parīkṣā* which gives it an exalted status in relation to other *siddhāntas*....

A commentator on the *Mānasa* (*Laghumānasa* of Muñjāla) has lamented: 'Indeed, the *siddhāntas*, like *Paitāmaha*, differ from one another [in giving the astronomical constants]. Timings are different as the *siddhāntas* differ (i.e. the measures of time for any particular event as computed by the different *siddhāntas* differ). When the computed timings differ, Vedic and domestic rituals, which have [correct] timings as a component [of their performance] go astray. When rituals go astray, worldly life gets disrupted. Alas, we have precipitated into a calamity.'

Here, it needs to be stated: 'O faint-hearted, there is nothing to be despaired of. Wherefore does anything remain beyond the ken of those intent on serving at the feet of the teachers? One has to realise that the five *siddhāntas* had been valid at a particular time. Therefore, one should look for a *siddhānta* that does not show discord with actual observations [at the present time]. Such accordance with observation has to be ascertained by astronomers during times of eclipses etc. When earlier *siddhāntas* show discord, observations should be made with instruments and revolutions etc. obtained, [which would give results which accord with actual observation] and a new *siddhānta* enunciated. Thus, nobody will be ridiculed in this world nor punished in the next'....

Therefore, the purpose of the *śāstra* is to create in students the capacity for examining the motion of the planets. They, in turn, should function in the world by composing a *karaṇa* [computational manual suitable for their epoch] which is in accordance with observations. Only such *karaṇas* can be accurate and of use in worldly affairs.